Let \( a \), \( b \), and \( x \) be natural variables. Variables \( a \) and \( b \) are implementer's variables, and \( x \) is a user's variable for the operations

\[
\begin{align*}
\text{start} & \equiv a := 0. \quad b := 0 \\
\text{step} & \equiv a := a + 1. \quad b := b + 2 \\
\text{ask} & \equiv x := a + b
\end{align*}
\]

Reimplement this theory replacing the two old implementer's variables \( a \) and \( b \) with one new natural implementer's variable \( c \).

(a) What is the data transformer?

(b) Using your data transformer, transform \( \text{step} \).

After trying the question, scroll down to the solution.
(a) What is the data transformer?
§
c = a + b

(b) Using your data transformer, transform step.
§
∀a, b · c = a+b ⇒ ∃a’, b’ · c’ = a’+b’ ∧ (a:= a+1.  b:= b+2) replace program
≡ ∀a, b · c = a+b ⇒ ∃a’, b’ · c’ = a’+b’ ∧ a’ = a+1 ∧ b’ = b+2 ∧ x’=x one-point
≡ ∀a, b · c = a+b ⇒ c’ = a+1+b+2 ∧ x’=x
≡ ∀a, b · c = a+b ⇒ c’ = a+b+3 ∧ x’=x one-point for b
≡ ∀a · c’ = a+c−a+3 ∧ x’=x
≡ ∀a’ · c’ = c+3 ∧ x’=x
≡ c’ = c+3 ∧ x’=x
≡ c:= c+3