456 Let a , b , and x be natural variables. Variables a and b are implementer's variables, and x is a user's variable for the operations

$$start \;=\; a:= 0.\; b:= 0$$
$$step \;=\; a:= a+1.\; b:= b+2$$
$$ask \;=\; x:= a+b$$

Reimplement this theory replacing the two old implementer's variables a and b with one new natural implementer's variable c .

(a) What is the data transformer?

(b) Using your data transformer, transform $step$.

After trying the question, scroll down to the solution.

(a) What is the data transformer?

§ $c = a+b$

(b) Using your data transformer, transform *step* .

§ $\forall a, b \cdot\ c = a+b \Rightarrow \exists a', b' \cdot\ c' = a'+b' \land (a{:=}\ a+1.\ \ b{:=}\ b+2)$ replace program

$=$ $\forall a, b \cdot\ c = a+b \Rightarrow \exists a', b' \cdot\ c' = a'+b' \land\ a' = a+1\ \land\ b' = b+2\ \land\ x'=x$ one-point

$=$ $\forall a, b \cdot\ c = a+b \Rightarrow\ c' = a+1+b+2\ \land\ x'=x$

$=$ $\forall a, b \cdot\ b = c-a \Rightarrow\ c' = a+b+3\ \land\ x'=x$ one-point for b

$=$ $\forall a \cdot\ c' = a+c-a+3\ \land\ x'=x$

$=$ $\forall a \cdot\ c' = c+3\ \land\ x'=x$

$=$ $c' = c+3\ \land\ x'=x$

$=$ $c{:=}\ c+3$