A theory provides three names: zero, increase, and inquire. It is presented by an implementation. Let \( u: \text{bin} \) be the user's variable, and let \( v: \text{nat} \) be the implementer's variable. The axioms are:

\[
\begin{align*}
\text{zero} &= v := 0 \\
\text{increase} &= v := v + 1 \\
\text{inquire} &= u := \text{even } v
\end{align*}
\]

Use data transformation to replace \( v \) with \( w: \text{bin} \) according to the transformer:

(a) \( w = \text{even } v \)

§ see book Section 7.2

(b) \( \top \)

§ Operation zero becomes

\[
\forall v: \top \implies \exists v': \top \land (v := 0)
\]

\[
= \forall v: \exists v'. u' = u \land v' = 0
\]

\[
= u' = u
\]

Operation increase becomes

\[
\forall v: \top \implies \exists v': \top \land (v := v + 1)
\]

\[
= \forall v: \exists v'. u' = u \land v' = v + 1
\]

\[
= u' = u
\]

Operation inquire becomes

\[
\begin{align*}
\forall v: \top &\implies \exists v': \top \land (u := \text{even } v) & \text{replace assignment and use identity law} \\
= \forall v: \exists v'. u' = \text{even } v \land v' = v & \text{one-point for } v' \\
= \forall v: u' = \text{even } v & \text{idempotent} \\
= (\forall v: u' = \text{even } v) \land (\forall v: u' = \text{even } v) & \text{specialize twice} \\
\implies u' = \text{even } 0 \land u' = \text{even } 1 \\
= u' = \top \land u' = \bot \\
= \bot
\end{align*}
\]

This transformer is so weak that inquire becomes unimplementable.

(c) \( \bot \) (this isn't a data transformer, since \( \forall w: \exists v: \bot \) isn't a theorem, but apply it anyway to see what happens)

§ Operation zero becomes

\[
\forall v: \bot \implies \exists v': \bot \land (v := 0)
\]

\[
= \top
\]

and the same for any other operation. This “transformer” is so strong that all operations become arbitrary (completely nondeterministic).