A theory provides three names: zero, increase, and inquire. It is presented by an implementation. Let \( u: \text{bin} \) be the user's variable, and let \( v: \text{nat} \) be the implementer's variable. The axioms are

\[
\begin{align*}
\text{zero} & \implies v := 0 \\
\text{increase} & \implies v := v + 1 \\
\text{inquire} & \implies u := \text{even } v
\end{align*}
\]

Use data transformation to replace \( v \) with \( w: \text{bin} \) according to the transformer

(a) \( \sqrt{w = \text{even } v} \)

(b) \( \top \)

(c) \( \bot \) (this isn't a data transformer, since \( \forall w \exists v \bot \) isn't a theorem, but apply it anyway to see what happens)

After trying the question, scroll down to the solution.
(a) \[ w = \text{even } v \]
§ see book Section 7.2
(b) \[ \top \]
§ Operation zero becomes
\[ \forall v \cdot \top \Rightarrow \exists v' \cdot \top \land (v' = 0) \]
\[ \equiv \forall v \cdot \exists v' \cdot u' = u \land v' = 0 \]
\[ \equiv u' = u \]
Operation increase becomes
\[ \forall v \cdot \top \Rightarrow \exists v' \cdot \top \land (v' = v+1) \]
\[ \equiv \forall v \cdot \exists v' \cdot u' = u \land v' = v+1 \]
\[ \equiv u' = u \]
Operation inquire becomes
\[ \forall v \cdot \top \Rightarrow \exists v' \cdot \top \land (u' = \text{even } v) \quad \text{replace assignment and use identity law} \]
\[ \equiv \forall v \cdot \exists v' \cdot u' = \text{even } v \land v' = v \quad \text{one-point for } v' \]
\[ \equiv \forall v \cdot u' = \text{even } v \quad \text{idempotent} \]
\[ \equiv (\forall v \cdot u' = \text{even } v) \land (\forall v \cdot u' = \text{even } v) \quad \text{specialize twice} \]
\[ \Rightarrow u' = \text{even } 0 \land u' = \text{even } 1 \]
\[ \equiv u' = \top \land u' = \bot \]
\[ \equiv \bot \]
This transformer is so weak that inquire becomes unimplementable.

(c) \[ \bot \] (this isn't a data transformer, since \[ \forall w \cdot \exists v' \cdot \bot \] isn't a theorem, but apply it anyway to see what happens)
§ Operation zero becomes
\[ \forall v \cdot \bot \Rightarrow \exists v' \cdot \bot \land (v' = 0) \]
\[ \equiv \top \]
and the same for any other operation. This “transformer” is so strong that all operations become arbitrary (completely nondeterministic).