449 (program list) A program list is a list with an associated index, and the following operations: *item* gives the value of the indexed item; *set x* changes the value of the indexed item to *x*; *goLeft* moves the index one item to the left; *goRight* moves the index one item to the right.

(a) Design axioms for a doubly infinite program list.

(b) Using your theory from part (a), prove

$$goLeft.\ set\ 3.\ goRight.\ set\ 4.\ goLeft \implies item'=3$$

After trying the question, scroll down to the solution.

(a) Design axioms for a doubly infinite program list.

§ Let L mean that all items to the left of the indexed item remain the same.
Let R mean that all items to the right of the indexed item remain the same.

$ok \;=\; L \wedge item'=item \wedge R \;=\; goLeft.\, goRight \;=\; goRight.\, goLeft$
$set\ x \;=\; L \wedge item'=x \wedge R$
$goLeft.\ L \wedge item'=item \;=\; L.\, goLeft$
$goRight.\ item'=item \wedge R \;=\; R.\, goRight$
$L.L \;=\; L$
$R.R \;=\; R$

(b) Using your theory from part (a), prove

$goLeft.\ set\ 3.\ goRight.\ set\ 4.\ goLeft \;\Longrightarrow\; item'=3$

§ $goLeft.\ set\ 3.\ goRight.\ set\ 4.\ goLeft$
$=\ goLeft.\ L \wedge item'=3 \wedge R.\ goRight.\ L \wedge item'=4 \wedge R.\ goLeft$
$\Longrightarrow\ goLeft.\ item'=3.\ goRight.\ L.\ goLeft$
$=\ goLeft.\ item'=3.\ goRight.\ goLeft.\ L \wedge item'=item$
$\Longrightarrow\ goLeft.\ item'=3.\ goRight.\ goLeft.\ item'=item$
$=\ goLeft.\ item'=3.\ item'=item$ definition of sequential composition twice
$=\ item'=3$