(insertion list) An insertion list is a data structure similar to a list, but with an associated insertion point.

\[
\begin{array}{cccccccccc}
\ldots & 4 & 7 & 1 & 0 & 3 & 8 & 9 & 2 & 5 & \ldots \\
\end{array}
\]

↑

**insert** puts an item at the insertion point (between two existing items), leaving the insertion point at its right. **erase** removes the item to the left of the insertion point, closing up the list. **item** gives the item to the left of the insertion point. **forward** moves the insertion point one item to the right. **back** moves the insertion point one item to the left.

(a) Design axioms for a doubly-infinite data-insertion list.
(b) Design axioms for a doubly-infinite program-insertion list.
(c) Design axioms for a finite data-insertion list.
(d) Design axioms for a finite program-insertion list.

After trying the question, scroll down to the solution.
(a) Design axioms for a doubly-infinite data-insertion list.

(b) Design axioms for a doubly-infinite program-insertion list.

§ Here is a weak theory.

\[
\begin{align*}
\text{item}' &= x \iff \text{insert } x \\
\text{item}' &= \text{item} \iff F \lor (\text{back.} \text{B.} \text{forward}) \\
\text{forward.} \text{back} &= \text{back.} \text{forward} = \text{ok} \\
F &= \text{ok} \lor (\exists x \cdot \text{insert } x) \lor \text{forward.} \text{erase} \lor \text{back} \lor (F \cdot F) \\
B &= \text{ok} \lor (\exists x \cdot \text{insert } x) \lor \text{erase} \lor (\text{back.} \text{B.} \text{forward}) \lor (B \cdot B)
\end{align*}
\]

Here is a strong theory.

\[
\begin{align*}
\text{ok} &= F \land B = \text{forward.} \text{back} = \text{back.} \text{forward} = \text{insert } x. \text{erase} \\
\text{insert } x &= (\text{back.} F) \land \text{item}' = x \land B \\
F &= \text{ok} \lor (\exists x \cdot \text{insert } x) \lor \text{forward.} \text{erase} \lor \text{back} \lor (F \cdot F) \\
B &= \text{ok} \lor (\exists x \cdot \text{insert } x) \lor \text{erase} \lor (\text{back.} \text{B.} \text{forward}) \lor (B \cdot B)
\end{align*}
\]

(c) Design axioms for a finite data-insertion list.

(d) Design axioms for a finite program-insertion list.