(semi-dependent composition) Independent composition \( P \parallel Q \) requires that \( P \) and \( Q \) have no variables in common, although each can make use of the initial values of the other's variables by making a private copy. In this question we explore another kind of composition, let's say semi-dependent composition \( P \parallel Q \). Like dependent composition, it requires \( P \) and \( Q \) to have the same variables. Like independent composition, it can be executed by executing the processes in parallel, but each process makes its assignments to local copies of variables. Then, when both processes are finished, the final value of a variable is determined as follows: if both processes left it unchanged, it is unchanged; if one process changed it and the other left it unchanged, its final value is the changed one; if both processes changed it, its final value is arbitrary. This final rewriting of variables does not require coordination or communication between the processes; each process rewrites those variables it has changed. In the case when both processes have changed a variable, we do not even require that the final value be one of the two changed values; the rewriting may mix the bits.

(a) Formally define semi-dependent composition, including time.

\[
\begin{align*}
P \parallel Q &= \exists x_0, x_0', y_0, y_0', ..., t_0, t_0' \\
(x_0', y_0', ..., t_0' \rightarrow P) x_0 y_0 ... t_0 \\
\land (x_0', y_0', ..., t_0' \rightarrow Q) x_0 y_0 ... t_0 \\
\land (x_0 = x' \Rightarrow x_0 = x') \land (x_0 = x' \Rightarrow x_0 = x') \\
\land (y_0 = y' \Rightarrow y_0 = y') \land (y_0 = y' \Rightarrow y_0 = y') \\
\land ... \\
\land t_0' = \text{max} \ t_0 \ t_0'
\end{align*}
\]

(b) What laws apply to semi-dependent composition?

(c) Under what circumstances is it unnecessary for a process to make private copies of variables?

(d) In variables \( x, y, \) and \( z \), without using \( \| \), express

\[
x := z \| y := z
\]

(e) In variables \( x, y, \) and \( z \), without using \( \| \), express

\[
x := y \| y := x
\]

(f) In variables \( x, y, \) and \( z \), without using \( \| \), express

\[
x := y \| x := z
\]

(g) In variables \( x, y, \) and \( z \), prove

\[
x := y \| x := z \quad \text{if} \ x = y \text{ then } x := z \text{ else if } x = z \text{ then } x := y \text{ else } x := y \| x := z \text{ fi fi}
\]

(h) In binary variables \( x, y, \) and \( z \), without using \( \| \), express

\[
x := x \land z \quad y := y \land \neg z \quad x := x \land \neg z \quad y := y \land z
\]

\[
x := x \land z \quad y := y \land \neg z \quad x := x \land \neg z \quad y := y \land z
\]

\[
\begin{align*}
\text{if } z \text{ then } x := x \| y := \bot \| x := \bot \| y := y \\
\text{else } x := \bot \| y := \bot \\
\end{align*}
\]

\[
\begin{align*}
x := \bot \| y := \bot \\
\neg x' \land \neg y' \land z' = z
\end{align*}
\]

(i) Let \( w : 0, 4 \) and \( z : 0, 1 \) be variables. Without using \( \| \), express

\[
w := 2 \times \text{max} (\text{div} \ w 2) \ z + \text{max} (\text{mod} \ w 2) (1-z)
\]

\[
w := 2 \times \text{max} (\text{div} \ w 2) (1-z) + \text{max} (\text{mod} \ w 2) \ z
\]
\[ w := 2 \times \text{max}(\text{div } w \ 2) \ z + \text{max}(\text{mod } w \ 2) \ (1-z) \]
\[ \quad \quad \quad \quad \text{|||} \quad w := 2 \times \text{max}(\text{div } w \ 2) \ (1-z) + \text{max}(\text{mod } w \ 2) \ z \]
\[ = \quad \quad \text{if } z=0 \ \text{then } w := 2 \times \text{div } w \ 2 + 1 \quad \text{|||} \quad w := 2 + \text{mod } w \ 2 \]
\[ \quad \quad \text{else } w := 2 + \text{mod } w \ 2 \quad \text{|||} \quad w := 2 \times \text{div } w \ 2 + 1 \ \text{fi} \]
\[ \quad \quad \text{is symmetric} \]
\[ = \quad \quad w := 2 \times \text{div } w \ 2 + 1 \quad \text{|||} \quad w := 2 + \text{mod } w \ 2 \]
\[ = \quad \text{if } w=0 \ \text{then } w := 1 \quad \text{|||} \quad w := 2 \]
\[ \quad \quad \text{else if } w=1 \ \text{then } w := 1 \quad \text{|||} \quad w := 3 \]
\[ \quad \quad \text{else if } w=2 \ \text{then } w := 3 \quad \text{|||} \quad w := 2 \]
\[ \quad \quad \text{else } w := 3 \quad \text{|||} \quad w := 3 \ \text{fi} \ \text{fi} \ \text{fi} \]
\[ = \quad (w\neq0 \Rightarrow w'=3) \ \land \ z'=z \]