

44 Let x be an element, and let A be any bunch. Prove $\neg x: A \iff \wp(A \dot{x}) = 0$ without using the law $\neg x: A \equiv \wp(A \dot{x}) = 0$.

After trying the question, scroll down to the solution.

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$$\begin{aligned} & \neg x: A \iff \wp(A'x) = 0 \\ = & x: A \implies \wp(A'x) \neq 0 \\ = & x = x'A \implies \wp(A'x) \neq 0 \\ = & x = A'x \implies \wp(A'x) \neq 0 \\ = & x = A'x \implies \wp x \neq 0 \\ = & x = A'x \implies 1 \neq 0 \\ = & x = A'x \implies \top \\ = & \top \end{aligned}$$

contrapositive
law $A: B = A = A'B$
law $A'B = B'A$
context
law $\wp x = 1$