A queue, according to our axioms, has an unlimited capacity to have items joined onto it. A limited-queue is a similar data structure but with a limited capacity to have items joined onto it.

(a) Design axioms for a limited-data-queue.
(b) Design axioms for a limited-program-queue.
(c) Can the limit be 0?
(a) Design axioms for a limited-data-queue.
§ I'm introducing new name \texttt{full}, which tells whether a queue is full (of course). This allows an implementation in which \texttt{full} might say \texttt{T} for a queue with 3 long items, and \texttt{⊥} for another queue with 3 short items in it. It also allows an implementation to allocate more space at any time, and deallocate unused space at any time. I also think it's the easiest solution.

\[
\begin{align*}
\text{emptyq}: \text{queue} & \Rightarrow \text{bin} \\
\text{full}: \text{queue} & \Rightarrow \text{bin} \\
\neg \text{full } q & \Rightarrow \text{join } q \times: \text{queue} \\
\neg \text{full } q & \Rightarrow \text{join } q \times \neq \text{emptyq} \\
\neg \text{full } q \land \neg \text{full } r & \Rightarrow (\text{join } q \times = \text{join } r \times \Rightarrow q=r \land x=y) \\
q+\text{emptyq} & \Rightarrow \text{leave } q: \text{queue} \\
q+\text{emptyq} & \Rightarrow \text{front } q: X \\
\neg \text{full } \text{emptyq} & \Rightarrow \text{leave } (\text{join } \text{emptyq} \times) = \text{emptyq} \\
q+\text{emptyq} \land \neg \text{full } q & \Rightarrow \text{leave } (\text{join } q \times) = \text{join } (\text{leave } q) \times \\
\neg \text{full } \text{emptyq} & \Rightarrow \text{front } (\text{join } \text{emptyq} \times) = \times \\
q+\text{emptyq} \land \neg \text{full } q & \Rightarrow \text{front } (\text{join } q \times) = \text{front } q
\end{align*}
\]

(b) Design axioms for a limited-program-queue.
§

\[
\begin{align*}
\text{mkemptyq} & \Rightarrow \text{isemptyq'} \\
\text{isemptyq} \land \neg \text{isfullq} \land \text{join } x & \Rightarrow \text{front'}=x \land \neg \text{isemptyq'} \\
\neg \text{isemptyq} \land \text{leave} & \Rightarrow \neg \text{isfullq'} \\
\neg \text{isemptyq} \land \neg \text{isfullq} \land \text{join } x & \Rightarrow \text{front'}=\text{front} \land \neg \text{isemptyq'} \\
\text{isemptyq} \land \neg \text{isfullq} & \Rightarrow (\text{join } x. \text{leave} = \text{mkemptyq}) \\
\neg \text{isemptyq} \land \neg \text{isfullq} & \Rightarrow (\text{join } x. \text{leave} = \text{leave}. \text{join } x)
\end{align*}
\]

(c) Can the limit be \texttt{0}?
§

The limit can be \texttt{0}. That happens in (a) when \texttt{full} is the constant \texttt{T} function; even \texttt{full emptyq} is \texttt{T}. In (b) it happens when \texttt{isfullq} is identically \texttt{T}.