Exercise 157 asks for a program to compute cumulative sums (running total). Write a program that can be transformed from sequential to parallel execution with $\log n$ time where $n$ is the length of the list.

Let $L$ be a list variable. The result we want is $R \ 0 \ (\#L)$ where

$$R\ ik \iff \#L'=\#L \land \forall \ j: \ i..k: \ L'j = \sum L [i..j+1]$$

That is the right form for a for-loop, but it will not be $\log$ time, and it will not be parallelizable. So we refine it as follows.

$$R\ ik \iff \begin{cases} \text{if } \#L \leq 1 \text{ then } \text{ok} \\ \text{else} \quad \text{var } m := \text{div} (k - i) 2. \\ \quad (R \ i \ m \| R \ m \ k). \\ \quad \text{for } j := m..k \text{ do } L'j := L'j + L \ (m-1) \text{ od} \ 
\end{cases}$$

The final for-loop can be parallelized, so that it takes no time. The computation then takes $\log (\#L)$ time.