A leafy tree is a tree with information residing only at the leaves. Design appropriate axioms for a binary leafy data-tree.

§ The following axioms constitute a strong theory of leafy trees.

\[ \text{leaf: } X \to \text{tree} \]
\[ \text{graft: } \text{tree} \to \text{tree} \to \text{tree} \]
\[ \text{leaf } X, \text{ graft } B B : B \Rightarrow \text{ tree: } B \]
\[ \text{graft } t u \neq \text{ leaf } x \]
\[ \text{leaf } x = \text{ leaf } y \iff x = y \]
\[ \text{graft } t u = \text{ graft } v w \iff t = v \land u = w \]
\[ \text{left (graft } t u ) = t \]
\[ \text{right (graft } t u ) = u \]
\[ \text{data (leaf } x \) = x \]

I have used a function \text{ leaf} to convert a data item to a one-item tree, and another function \text{ data} to retrieve it again. Another, simpler, approach is to consider that a data item is already a one-item tree. In that case, \text{ leaf} and \text{ data} aren't needed. The axioms are:

\[ \text{X: tree} \]
\[ \text{graft: } \text{tree} \to \text{tree} \to \text{tree} \]
\[ X, \text{ graft } B B : B \Rightarrow \text{ tree: } B \]
\[ \neg \text{ graft } t u : X \]
\[ \text{graft } t u = \text{ graft } v w \iff t = v \land u = w \]
\[ \text{left (graft } t u ) = t \]
\[ \text{right (graft } t u ) = u \]