In general, each node of a tree may have any number of subtrees.

(a) Design a data theory for general trees.
(b) Implement your theory.
(c) Prove your implementation.

After trying the question, scroll down to the solution.
(a) Design a data theory for general trees.

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\[ \text{tree} \neq \text{null} \]
\[ L: [*\text{tree}] \land x: X \Rightarrow \text{construct} \ L x: \text{tree} \]
\[ L: [*\text{tree}] \land x: X \Rightarrow \text{root} (\text{construct} \ L x) = x \]
\[ L: [*\text{tree}] \land x: X \land n \neq \#L \Rightarrow \text{child} n (\text{construct} \ L x) = L n \]

If we also want an empty tree, we can add the axioms

\[ \text{emptree}: \text{tree} \]
\[ \text{emptree} \neq \text{construct} \ L x \]

If we want an arity function, we add

\[ L: [*\text{tree}] \land x: X \Rightarrow \text{arity} (\text{construct} \ L x) = \#L \]

(b) Implement your theory.

(c) Prove your implementation.