In general, each node of a tree may have any number of subtrees.

(a) Design a data theory for general trees.

\[
\begin{align*}
\text{tree} & \neq \text{null} \\
L: [\text{tree}] \land x: X & \Rightarrow \text{construct } L x: \text{tree} \\
L: [\text{tree}] \land x: X & \Rightarrow \text{root } (\text{construct } L x) = x \\
L: [\text{tree}] \land x: X \land n < \#L & \Rightarrow \text{child } n (\text{construct } L x) = L n
\end{align*}
\]

If we also want an empty tree, we can add the axioms

\[
\text{emptree: tree} \\
\text{emptree} \neq \text{construct } L x
\]

If we want an arity function, we add

\[
L: [\text{tree}] \land x: X \Rightarrow \text{arity } (\text{construct } L x) = \#L
\]

(b) Implement your theory.

(c) Prove your implementation.