

431 Implement weak program-stack theory (Subsection 7.1.3) as follows: the implementer's variable is a list that grows and never shrinks. A popped item must be marked as garbage.

After trying the question, scroll down to the solution.

§ Except for the requirement that “A popped item must be marked as garbage.”, here is a wasteful but correct solution. Leave *push* and *top* alone, and redefine

$$\text{pop} = s := s - 1 ; s[0;..s-1]$$

Here is an efficient solution. The implementer's variables are  $L: [*X]$  and  $s: \text{nat}$ . Items  $L i$  for  $s \leq i$  are garbage, so  $s$  marks the start of the garbage.

$$\text{push} = \langle x: X \cdot \mathbf{if} \#L=s \mathbf{then} L := L ; [x] \mathbf{else} L := s \rightarrow x \mid L. s := s+1 \mathbf{fi} \rangle$$

$$\text{pop} = s := s - 1$$

$$\text{top} = L(s-1)$$

$$\text{balance} = s' = s \wedge \forall i: 0, \dots, s. L' i = L i$$

We needed to define *balance* but not to implement it. Now we need to prove the axioms. All proofs proceed by substituting the definitions into the axioms and then using list theory.