An old implementer's variable \( c: -1, 0, 1 \) is being replaced by new implementer's variables \( a, b: \text{bin} \) such that \( c=-1 \) is replaced by \( a \) and \( b \) both being \( \bot \), \( c=1 \) is replaced by \( a \) and \( b \) both being \( \top \), and \( c=0 \) is replaced by \( a \) and \( b \) being unequal.

(a) What is the transformer?
§
\[
c = -1 \land \neg a \land \neg b \\
\lor \ c = 0 \land a \lor b \\
\lor \ c = 1 \land a \land b
\]
This is a transformer because
\[
\forall a, b. \exists c. \ c = -1 \land \neg a \land \neg b \lor c = 0 \land a \lor b \lor c = 1 \land a \land b
\]
\[
\Rightarrow \exists c'. (c' = -1 \land \neg a' \land \neg b' \lor c' = 0 \land a' \lor b' \lor c' = 1 \land a' \land b') \land c' = 0 \quad \text{one-point}
\]
\[
\Rightarrow 0 = -1 \land \neg a' \land \neg b' \lor 0 = 0 \land a' \lor b' \lor 0 = 1 \land a' \land b' \quad \text{arithmetic, base, identity}
\]
\[
\Rightarrow a' \land \neg b' \quad \text{antidistributive law}
\]
\[
\Rightarrow (\exists c. \ c = -1 \land \neg a \land \neg b \lor c = 0 \land a \lor b \lor c = 1 \land a \land b)
\]
\[
\Rightarrow a' \land \neg b' \quad \text{above proof that we have a transformer}
\]
\[
\Rightarrow \top \Rightarrow a' \land \neg b' \quad \text{identity}
\]
\[
\Rightarrow a' \land \neg b' \leftarrow a := \top. \ b := \bot
\]