We defined bunch \textit{null} with the axiom \textit{null} \(: A \). Is there any harm in defining bunch \textit{all} with the axiom \textit{A} \(: \text{all} \)?

§

With just Binary Theory, Number Theory, Character Theory, and Bunch Theory, there is no harm (inconsistency) in defining \textit{all} with the axiom \textit{A} \(: \text{all} \). Even when we add Set Theory (in this book; we don't yet have set comprehension) there is no harm. But when we add Function Theory, specifically the \(\$\) quantifier, we have an inconsistency known as “Russell's Paradox”. Let

\[ R = \{s: \forall s \rightarrow s \in s\} \]

Then \( R \) is the set of all sets that are not members of themselves. Or, without abbreviation,

\[ R = \{s: \forall s \rightarrow s \in s\} \]

Then

\[
\begin{align*}
R \in R & \quad \text{definition of } R \\
\equiv & \quad R \in \{s: \forall s \rightarrow s \in s\} \quad \text{\(\in\) axiom} \\
\equiv & \quad R: \forall s: \forall s \rightarrow s \in s \quad \text{solution law} \\
\equiv & \quad R: \forall s \quad \text{definition of } R \\
\equiv & \quad \{s: \forall s \rightarrow s \in s\}: \forall R \quad \forall \text{ axiom} \\
\equiv & \quad (\forall s: \forall s \rightarrow s \in s): \forall R \quad \text{all axiom} \\
\equiv & \quad \top \quad \text{identity law} \\
\equiv & \quad \neg R \in R
\end{align*}
\]

and we have inconsistency.

It might be nice to have \textit{all}, and to weaken the solution law to accommodate it. But I have stayed with standard mathematics, excluding \textit{all} and including the strong form of solution law.

Even without \textit{all}, we still have a benign form of Russell's Paradox (Exercise 48); it is not an inconsistency, but it may disturb some people.