427 In data-queue theory (Subsection 7.0.3), prove that if you start with an empty queue, and join two items, the first item joined is the front of the queue.

After trying the question, scroll down to the solution.

§ Here is data-queue theory.

(0) *emptyq*: *queue*
(1) *join q x*: *queue*
(2) *join q x* ≠ *emptyq*
(3) *join q x* = *join r y* = *q*=*r* ∧ *x*=*y*
(4) *emptyq*, *join B X*: *B* ⇒ *queue*: *B*
(5) *leave* (*join emptyq x*) = *emptyq*
(6) *q*≠*emptyq* ⇒ *leave* (*join q x*) = *join* (*leave q*) *x*
(7) *front* (*join emptyq x*) = *x*
(8) *q*≠*emptyq* ⇒ *front* (*join q x*) = *front q*

⊤ instantiate (8) with *join emptyq x* for *q* and *y* for *x*
= *join emptyq x* ≠ *emptyq* ⇒ *front* (*join* (*join emptyq x*) *y*) = *front join* (*emptyq x*)
use (2) with *emptyq* for *q*
= ⊤ ⇒ *front* (*join* (*join emptyq x*) *y*) = *front* (*join emptyq x*) identity
= *front* (*join* (*join emptyq x*) *y*) = *front* (*join emptyq x*) use (7) on the right side
= *front* (*join* (*join emptyq x*) *y*) = *x*