426 (data-queue implementation) Implement the data-queue theory presented in Section 7.0.

After trying the question, scroll down to the solution.

§ The theory to be implemented is:

$\mathit{emptyq}: \mathit{queue}$

$\mathit{join}\ q\ x: \mathit{queue}$

$\mathit{join}\ q\ x \neq \mathit{emptyq}$

$\mathit{join}\ q\ x = \mathit{join}\ r\ y \quad = \quad q{=}r \land x{=}y$

$q{\neq}\mathit{emptyq} \quad \Rightarrow \quad \mathit{leave}\ q: \mathit{queue}$

$q{\neq}\mathit{emptyq} \quad \Rightarrow \quad \mathit{front}\ q: X$

$\mathit{leave}\ (\mathit{join}\ \mathit{emptyq}\ x) \ = \ \mathit{emptyq}$

$\mathit{front}\ (\mathit{join}\ \mathit{emptyq}\ x) \ = \ x$

$q{\neq}\mathit{emptyq} \quad \Rightarrow \quad \mathit{leave}\ (\mathit{join}\ q\ x) = \mathit{join}\ (\mathit{leave}\ q)\ x$

$q{\neq}\mathit{emptyq} \quad \Rightarrow \quad \mathit{front}\ (\mathit{join}\ q\ x) = \mathit{front}\ \ q$

And here's an implementation.

$\mathit{queue} \ = \ [*X]$

$\mathit{emptyq} \ = \ [\mathit{nil}]$

$\mathit{join} \ = \ \langle q: \mathit{queue}\cdot \langle x: X\cdot q;;[x]\rangle\rangle$

$\mathit{leave} \ = \ \langle q: \mathit{queue}\cdot q[1;..\#q]\rangle$

$\mathit{front} \ = \ \langle q: \mathit{queue}\cdot q\ 0\rangle$

All the axioms are proved by substituting the implementations and then using list theory.