Given natural number $n$, a theory maintains a subbunch of $0,..,n$. The operations are: \textit{mkempty}, which makes the bunch empty; \textit{insert} $x$, which inserts $x$ into the bunch; \textit{remove} $x$, which removes $x$ if it was there, and \textit{check} $x$ which tells whether $x$ is there by assigning to a user's binary variable $u$.

(a) Design axioms that are weak enough to allow other operations to be added to the theory. § The axioms use an auxiliary operation \textit{preserve} $x$ that does not affect whether $x$ is in the bunch.

\begin{align*}
\text{mkempty.} & \quad \text{preserve } x. \quad \text{check } x \Rightarrow \neg u' \\
\text{insert } x. & \quad \text{preserve } x. \quad \text{check } x \Rightarrow u' \\
\text{remove } x. & \quad \text{preserve } x. \quad \text{check } x \Rightarrow \neg u' \\
\text{preserve } x & \Leftarrow \text{ok} \\
\text{preserve } x & \Leftarrow \text{check } x \\
\text{preserve } x & \Leftarrow \text{insert } y \land x \neq y \\
\text{preserve } x & \Leftarrow \text{remove } y \land x \neq y \\
\text{preserve } x & \Leftarrow \text{preserve } x. \quad \text{preserve } x
\end{align*}

(b) Implement your theory of part (a) as a list of binary values.

(c) Transform your implementation of part (b) to one that maintains a list of natural numbers.