Let \( p \) be a user's binary variable, and let \( m \) be an implementer's natural variable. The operations allow the user to assign a value \( n \) to the implementer's variable, and to test whether the implementer's variable is a prime number.

\[
\begin{align*}
\text{assign } n &= m := n \\
\text{check } &= p := \text{prime } m
\end{align*}
\]

assuming \textit{prime} is suitably defined. If \textit{prime} is an expensive function, and the \textit{check} operation is more frequent than the \textit{assign} operation, we can improve the solution by making \textit{check} less expensive even if that makes \textit{assign} more expensive. Using data transformation, make this improvement.

§ I replace the implementer's natural variable \( m \) by a new implementer's binary variable \( q \). The data transformer is

\[
q = \text{prime } m
\]

We have to check that this is a data transformer.

\[
\forall q \exists m : q = \text{prime } m
\]

Using this transformer, \textit{assign }\( n \) is transformed to

\[
\begin{align*}
\forall m : q &= \text{prime } m \Rightarrow \exists m' : q' = \text{prime } m' \land (m := n) \\
&= \forall m : q = \text{prime } m \Rightarrow \exists m' : q' = \text{prime } m' \land m' = n \land p' = p & \text{expand assignment} \\
&= \forall m : q = \text{prime } m \Rightarrow q' = \text{prime } n \land p' = p & \text{one-point } m' \\
&= \forall r : \text{prime nat} : q=r \Rightarrow q' = \text{prime } n \land p' = p & \text{change of variable from } m \text{ to } r \\
&= q' = \text{prime } n \land p' = p & \text{one-point } r \\
&= q := \text{prime } n
\end{align*}
\]

Using this transformer, \textit{check }\( n \) is transformed to

\[
\begin{align*}
\forall m : q &= \text{prime } m \Rightarrow \exists m' : q' = \text{prime } m' \land (p := \text{prime } m) \\
&= \forall m : q = \text{prime } m \Rightarrow \exists m' : q' = \text{prime } m' \land m' = m \land p' = \text{prime } m & \text{expand assignment} \\
&= \forall m : q = \text{prime } m \Rightarrow q' = \text{prime } m \land p' = \text{prime } m & \text{one-point } m' \\
&= \forall r : \text{prime nat} : q=r \Rightarrow q' = r \land p' = r & \text{change of variable from } m \text{ to } r \\
&= q' = q \land p' = q & \text{one-point } r \\
&= p := q
\end{align*}
\]