In real variable $x$, consider the equation $P \equiv P. x := x^2$

(a) Find 7 distinct solutions for $P$.

§ Here are 6 solutions: $x' = 0$; $x' > 0$; $0 < x' < 1$; $x' = 1$; $x' > 1$; $\bot$. The disjunction of any two solutions is also a solution. For any binary expression $b$ and solutions $A$ and $B$, $\text{if } b \text{ then } A \text{ else } B \text{ fi}$ is also a solution.

(b) Which solution does recursive construction give starting from $\top$? Is it the weakest solution?

§ $P_0 \equiv \top$

$P_1 \equiv P_0. \; x := x^2$

$\equiv \top. \; x' = x^2$

$\equiv \exists x''. \; \top \land x' = x''^2$

$\equiv \exists x''. \; x' = x''^2$

I don't have a law to quote here, but here's my reasoning.

If $x''$ is any real value, its square is nonnegative.

$\equiv x' \geq 0$

It gives $x' \geq 0$, which is the weakest solution.

(c) If we add a time variable, which solution does recursive construction give starting from $t' \geq t$? Is it a strongest implementable solution?

§ It gives $t' = \infty \land x' \geq 0$, which is not a strongest implementable solution because $t' = \infty \land x' = 0$ is a stronger implementable solution.

(d) Now let $x$ be an integer variable, and redo the question.

§ The solutions are: $x' = 0$; $x' = 1$; $\bot$; the disjunction of any two solutions is also a solution; for any binary expression $b$ and solutions $A$ and $B$, $\text{if } b \text{ then } A \text{ else } B \text{ fi}$ is also a solution. Starting from $\top$ we get $x' = 0 \lor x' = 1$ which is the weakest solution. Starting from $t' \geq t$ we get $t' = \infty \land (x' = 0 \lor x' = 1)$ which is not a strongest implementable solution because $t' = \infty \land x' = 0$ is a stronger implementable solution.