The user's variable is binary \( b \). The implementer's variables are natural \( x \) and \( y \). The operations are:

\[
\begin{align*}
\text{done} & \Rightarrow b := x = y = 0 \\
\text{step} & \Rightarrow \begin{cases} y > 0 \text{ then } y := y - 1 & \text{else } x := x - 1 \end{cases}.
\end{align*}
\]

\var n: \text{nat} \ y := n \text{ fi}

Replace the two implementer's variables \( x \) and \( y \) with a single new implementer's variable: natural \( z \).

§

Use transformer \( x = z \land y = 0 \). Then done becomes

\[
\forall x, y. x = z \land y = 0 \Rightarrow \exists x', y'. x' = z' \land y' = 0 \land b' = (x = y = 0) \land x' = x \land y' = y \text{ one-pt } x, y
\]

and step becomes

\[
\forall x, y. x = z \land y = 0 \Rightarrow \exists x', y'. x' = z' \land y' = 0 \land \begin{cases} y > 0 \text{ then } y := y - 1 & \text{else } x := x - 1 \end{cases}.
\]

Or, use transformer \( z = x + y \). Then done becomes

\[
\forall x, y. z = x + y \Rightarrow \exists x', y'. x' = z' \land y' = 0 \land (y > 0 \Rightarrow x' = x \land y' = y - 1) \land (y = 0 \Rightarrow x' = x - 1)
\]

and step becomes

\[
\forall x, y. z = x + y \Rightarrow \exists x', y'. x' = z' \land y' = 0 \land x' = z - 1 \text{ one-pt } x', y'
\]

The needed lemma, that every natural \( z \) is the sum of two naturals, is proved as follows:

\[
\exists x, y. z = x + y \text{ generalization: for } x \text{ use } z \text{ and for } y \text{ use } 0
\]

\(\therefore z = z + 0\)

\(\therefore z = z + 0\)

\(\therefore T\)

\(\therefore T\)

For the right conjunct, use context \( y = 0 \) to simplify \( z = x + y \), and then one-pt on \( x \) and \( y \)

\[
\begin{align*}
\forall x, y. z = x + y \land y > 0 & \Rightarrow z' = x + y - 1 \land z' \geq z - 1 \\
\forall x, y. z = x + y \land y > 0 & \Rightarrow z' = x + y - 1 \land z' = z - 1 \\
\forall x, y. z = x + y \land y > 0 & \Rightarrow z' = x + y - 1 \land z' = z - 1 \\
\forall x, y. z = x + y \land y > 0 & \Rightarrow z' = x + y - 1 \land z' \geq z - 1
\end{align*}
\]
Or, taking a hint from Exercise 300, which is solved in Chapter 5, we could let $f: \text{nat} \to \text{nat}$ be an unknown function, let $s = \Sigma f [0..x]$, and use transformer $z = x + y + s$. 