A theory provides three names: \textit{zero}, \textit{increase}, and \textit{inquire}. It is presented by an implementation. Let \( u : \text{bin} \) be the user's variable, and let \( v : \text{nat} \) be the implementer's variable. The axioms are

\[
\begin{align*}
\text{zero} & \equiv v := 0 \\
\text{increase} & \equiv v := v + 1 \\
\text{inquire} & \equiv u := \text{even } v
\end{align*}
\]

Use data transformation to replace \( v \) with \( w : \text{bin} \) according to the transformer

(a) \( w = \text{even } v \)

(b) \( T\)

§ Operation \textit{zero} becomes

\[
\forall v'. T \Rightarrow \exists v'. T \land (v := 0)
\]

\[= \forall v'. \exists v'. u' = u \land v' = 0\]

\[= u' = u\]

Operation \textit{increase} becomes

\[
\forall v'. T \Rightarrow \exists v'. T \land (v := v + 1)
\]

\[= \forall v'. \exists v'. u' = u \land v' = v + 1\]

\[= u' = u\]

Operation \textit{inquire} becomes

\[
\forall v'. T \Rightarrow \exists v'. T \land (u := \text{even } v)
\]

\[= \forall v'. \exists v'. u' = \text{even } v \land v' = v\]

\[= \perp\]

This transformer is so weak that \textit{inquire} becomes unimplementable.

(c) \( \perp \) (this isn't a data transformer, since \( \forall w'. \exists v'. \perp \) isn't a theorem, but apply it anyway to see what happens)

§ Operation \textit{zero} becomes

\[
\forall v'. \perp \Rightarrow \exists v'. \perp \land (v := 0)
\]

\[= T\]

and the same for any other operation. This “transformer” is so strong that all operations become arbitrary (completely nondeterministic).