A theory provides three names: zero, increase, and inquire. It is presented by an implementation. Let $u : \text{bin}$ be the user's variable, and let $v : \text{nat}$ be the implementer's variable. The axioms are:

- $\text{zero} = v := 0$
- $\text{increase} = v := v + 1$
- $\text{inquire} = u := \text{even } v$

Use data transformation to replace $v$ with $w : \text{bin}$ according to the transformer:

(a) $w = \text{even } v$

(b) $\top$

§ Operation zero becomes

- $\forall v : \top \Rightarrow \exists v' : \top \land (v := 0)$
- $= \forall v : \top \exists v' : u' = u \land v' = 0$
- $= u' = u$

Operation increase becomes

- $\forall v : \top \Rightarrow \exists v' : \top \land (v := v + 1)$
- $= \forall v : \top \exists v' : u' = u \land v' = v + 1$
- $= u' = u$

Operation inquire becomes

- $\forall v : \top \Rightarrow \exists v' : \top \land (u := \text{even } v)$
- $= \forall v : \top \exists v' : u' = \text{even } v \land v' = v$
- $= \forall v : \top u' = \text{even } v$
- $= (\forall v : u' = \text{even } v) \land (\forall v : u' = \text{even } v)$
- $= u' = \text{even } 0 \land u' = \text{even } 1$
- $= u' = \top \land u' = \bot$
- $= \bot$

This transformer is so weak that inquire becomes unimplementable.

(c) $\bot$ (this isn't a data transformer, since $\forall w : \top \exists v : \bot$ isn't a theorem, but apply it anyway to see what happens)

§ Operation zero becomes

- $\forall v : \bot \Rightarrow \exists v' : \bot \land (v := 0)$
- $= \top$

and the same for any other operation. This “transformer” is so strong that all operations become arbitrary (completely nondeterministic).