Let \( x \) be an integer variable.

(a) Using the recursive time measure, add time and then find the strongest implementable specification \( S \) that you can find for which:

\[
S \iff \begin{cases} 
  x=0 & \text{then } ok \\
  x>0 & \text{then } x:= x-1. \quad S \\
  x' \geq 0 & \text{fi fi}
\end{cases}
\]

Assume that \( x' \geq 0 \) takes no time.

(b) What do we get from recursive construction starting with \( t' \geq t \) ?

After trying the question, scroll down to the solution.
(a) Using the recursive time measure, add time and then find the strongest implementable specification \( S \) that you can find for which

\[
S \Leftarrow \begin{cases} 
\text{if } x = 0 & \text{then } \text{ok} \\
\text{else if } x > 0 & \text{then } x := x - 1. \ S \\
\text{else } x' \geq 0. \ S \end{cases}
\]

Assume that \( x' \geq 0 \) takes no time.

§ Adding time,
\[
S \Leftarrow \begin{cases} 
\text{if } x = 0 & \text{then } \text{ok} \\
\text{else if } x > 0 & \text{then } x := x - 1. \ t := t + 1. \ S \\
\text{else } x' \geq 0 \land t' = t. \ t := t + 1. \ S \end{cases}
\]

the strongest implementable solution for \( S \) is
\[
x' = 0 \land \begin{cases} 
\text{if } x = 0 & t' = t + x \text{ else } t' \geq t + 1 \end{cases}
\]

If we replace \( x' \geq 0 \land t' = t \) by \( x := c \) where \( c \) is an arbitrary natural number, then we can prove
\[
x' = 0 \land \begin{cases} 
\text{if } x \geq 0 & t' = t + x \text{ else } t' = t + 1 + c \end{cases}
\]

(b) What do we get from recursive construction starting with \( t' \geq t \)?

§
\[
S_n = \begin{cases} 
0 \leq x < n \land x' = 0 \land t' = t + x \\
\lor \begin{cases} 
\neg 0 \leq x < n \land t' \geq t + n \\
x < 0 \land x' = 0 \land t + 1 \leq t' < t + n 
\end{cases}
\end{cases}
\]
\[
S_{\infty} = \begin{cases} 
0 \leq x \land x' = 0 \land t' = t + x \\
\lor \begin{cases} 
x < 0 \land t' = \infty \\
x < 0 \land x' = 0 \land t + 1 \leq t' < \infty 
\end{cases}
\end{cases}
\]

\( S_{\infty} \) is a solution to the given implication, but not as strong as the solution shown in part (a). It is interesting to note that if the given implication were an equation, then \( S_{\infty} \) would not be a solution (fixed-point), but the solution of part (a) would still be a solution.

§ Note: \( S_n \) is the same as \( S_{\infty} \).