Let $x$ be an integer variable.

(a) Using the recursive time measure, add time and then find the strongest implementable specification $S$ that you can find for which

\[
S \iff \begin{cases} 
    x=0 & \text{then} \ ok \\
    x>0 & \text{then} \ x:=x-1. \ S \\
    x' \geq 0 & \text{else} \ S \text{ fi fi}
\end{cases}
\]

Assume that $x' \geq 0$ takes no time.

Adding time,

\[
S \iff \begin{cases} 
    x=0 & \text{then} \ ok \\
    x>0 & \text{then} \ x:=x-1. \ t:=t+1. \ S \\
    x' \geq 0 \land t'=t. \ t:=t+1. \ S \text{ fi fi}
\end{cases}
\]

the strongest implementable solution for $S$ is

\[
x'=0 \land \begin{cases} 
    x \geq 0 & \text{then} \ t'=t+x \text{ else} \ t' \geq t+1 \text{ fi}
\end{cases}
\]

If we replace $x' \geq 0 \land t'=t$ by $x:=c$ where $c$ is an arbitrary natural number, then we can prove

\[
x'=0 \land \begin{cases} 
    x \geq 0 & \text{then} \ t'=t+x \text{ else} \ t'=t+1+c \text{ fi}
\end{cases}
\]

(b) What do we get from recursive construction starting with $t' \geq t$ ?

\[
\begin{align*}
S_n &= 0 \leq x<n \land x'=0 \land t'=t+x \\
     &\lor 0 \leq x<n \land t' \geq t+n \\
     &\lor x<0 \land x'=0 \land t+1 \leq t'<t+n \\
S_\infty &= 0 \leq x \land x'=0 \land t'=t+x \\
      &\lor x<0 \land t'=\infty \\
      &\lor x<0 \land x'=0 \land t+1 \leq t'<\infty
\end{align*}
\]

$S_\infty$ is a solution to the given implication, but not as strong as the solution shown in part (a). It is interesting to note that if the given implication were an equation, then $S_\infty$ would not be a solution (fixed-point), but the solution of part (a) would still be a solution.

\[n \ S_n \text{ is the same as } S_\infty.\]