Section 6.1 defines program \( \text{zap} \) by the fixed-point equation

\[
\text{zap} \equiv \begin{cases} 
\text{if } x = 0 & \text{then } y := 0 \\
\text{else } x := x - 1. \quad t := t + 1. \quad \text{zap} \end{cases}
\]

(a) What axiom is needed to make \( \text{zap} \) the weakest fixed-point?

(b) What axiom is needed to make \( \text{zap} \) the strongest fixed-point?

(c) Section 6.1 gives six solutions to this equation. Find more solutions. Hint: strange things can happen at time \( \infty \).

After trying the question, scroll down to the solution.
(a) What axiom is needed to make $\text{zap}$ the weakest fixed-point?

§ $(\forall \sigma, \sigma' \cdot Z = \text{if } x=0 \text{ then } y:= 0 \text{ else } x:= x-1. \ t:= t+1. \ Z \text{ fi}) \Rightarrow (\forall \sigma, \sigma' \cdot \text{zap} \Leftarrow Z)$

(b) What axiom is needed to make $\text{zap}$ the strongest fixed-point?

§ $(\forall \sigma, \sigma' \cdot Z = \text{if } x=0 \text{ then } y:= 0 \text{ else } x:= x-1. \ t:= t+1. \ Z \text{ fi}) \Rightarrow (\forall \sigma, \sigma' \cdot Z \Leftarrow \text{zap})$

(c) Section 6.1 gives six solutions to this equation. Find more solutions. Hint: strange things can happen at time $\infty$.

§ $\text{if } x\geq 0 \text{ then } x'=y'=0 \land t' = t+x \text{ else } x'=y'=17 \land t' = \infty \text{ fi}$