400 Define language $\text{lang}$ by the fixed-point construction axiom

$$
\text{lang} = \text{nil}, \text{"\textquotedblleft}; \text{lang}; \text{"\textquotedblright"}, \text{lang};\text{lang}
$$

and associated fixed-point induction axiom.

(a) Informally, what is the language described?

(b) Write an equivalent, nonrecursive definition of the language. Hint: start with § and use a predicate that counts occurrences of characters in a text.

After trying the question, scroll down to the solution.

(a) Informally, what is the language described?

§ The language of matching brackets.

(b) Write an equivalent, nonrecursive definition of the language. Hint: start with § and use a predicate that counts occurrences of characters in a text.

§ Define

$$
\begin{align*}
\text{brackets} & = \text{"\textquotedblleft}; \text{\textquotedblright"} \\
\text{strings} & = *\text{brackets} \\
\text{occ} & = \langle c: \text{brackets}; s: \text{strings}; \varphi s: 0,..\leftrightarrow s; s=c \rangle
\end{align*}
$$

Then the language is

$$
\begin{align*}
\text{strings}; \text{occ \"\textquotedblleft}; s = \text{occ \textquotedblright\"}; s & \land \forall i: 0,..\leftrightarrow s; \text{occ \"\textquotedblleft}; s_{0..i} \geq \text{occ \textquotedblright\"}; s_{0..i}
\end{align*}
$$