Truth tables and the Evaluation Rule can be replaced by some new axioms and antiaxioms. For example, one truth table entry becomes the axiom $\top \lor \top$ and another becomes the axiom $\top \lor \bot$. These two axioms can be reduced to one axiom by the introduction of a variable, giving $\top \lor x$. Write the truth tables as axioms and antiaxioms as succinctly as possible.

After trying the question, scroll down to the solution.
Writing the truth tables as axioms and antiaxioms is easy: one axiom for each \( \top \) entry, and one antiaxiom for each \( \bot \) entry. However, in preparation for the next step, I'll use the Consistency Rule to write the antiaxioms as axioms by starting with a \( \neg \) sign. Here they are in order of their appearance on pages 3 and 4.

\[
\begin{align*}
\neg\neg \top & \quad \bot \Rightarrow \top \\
\neg \bot & \quad \bot \Rightarrow \bot \\
\top \land \top & \quad T \Leftarrow \top \\
\neg(\bot \land \top) & \quad T \Leftarrow \bot \\
\neg(\bot \land \bot) & \quad \bot \Leftarrow \bot \\
\top \lor \bot & \quad \top \Leftarrow x \\
\bot \lor \top & \quad \bot \Leftarrow \bot \\
\neg(\bot \lor \bot) & \quad \bot \Leftarrow \bot \\
\top \Rightarrow \bot & \quad \bot \Leftarrow \bot \\
\neg(\top \Rightarrow \bot) & \quad \bot \Leftarrow \bot \\
\bot \Rightarrow \top & \quad \neg(\top \Rightarrow \bot) \\
\neg(\top \Rightarrow \bot) & \quad \bot \Leftarrow \bot \\
\top \Rightarrow \bot & \quad \bot \Leftarrow \bot \\
\neg(\bot \Rightarrow \bot) & \quad \bot \Leftarrow \bot \\
\end{align*}
\]

Now I use the Completion and Instance Rules to pair axioms that differ in only one position. An axiom can participate in more than one pairing.

\[
\begin{align*}
\neg\neg \top & \quad x \Rightarrow \top \\
\neg \bot & \quad \neg(\bot \Rightarrow \bot) \\
\top \land \top & \quad \top \Leftarrow x \\
\neg(\bot \land \bot) & \quad \bot \Leftarrow \bot \\
\neg(\bot \land \bot) & \quad \bot \Leftarrow \bot \\
\top \lor \bot & \quad \top \Leftarrow x \\
\bot \lor \top & \quad \bot \Leftarrow \bot \\
\neg(\bot \lor \bot) & \quad \bot \Leftarrow \bot \\
\end{align*}
\]

It may seem that we can use symmetry to make the list even shorter. But the symmetry laws are proven from these axioms, so we can't.