4 Value tables and the Evaluation Rule can be replaced by some new axioms and antiaxioms. For example, one value table entry becomes the axiom $\top \lor \top$ and another becomes the axiom $\top \lor \bot$. These two axioms can be reduced to one axiom by the introduction of a variable, giving $\top \lor x$. Write the value tables as axioms and antiaxioms as succinctly as possible.

After trying the question, scroll down to the solution.

§ Writing the value tables as axioms and antiaxioms is easy: one axiom for each $\top$ entry, and one antiaxiom for each $\bot$ entry. However, in preparation for the next step, I'll use the Consistency Rule to write the antiaxioms as axioms by starting with a $\neg$ sign. Here they are in order of their appearance on pages 3 and 4.

$\neg\neg\top$	$\bot\Rightarrow\top$	$\bot\ne\top$
$\neg\bot$	$\bot\Rightarrow\bot$	$\neg(\bot\ne\bot)$
$\top\wedge\top$	$\top\Leftarrow\top$	**if** $\top$ **then** $\top$ **else** $\top$ **fi**
$\neg(\top\wedge\bot)$	$\top\Leftarrow\bot$	**if** $\top$ **then** $\top$ **else** $\bot$ **fi**
$\neg(\bot\wedge\top)$	$\neg(\bot\Leftarrow\top)$	$\neg$ **if** $\top$ **then** $\bot$ **else** $\top$ **fi**
$\neg(\bot\wedge\bot)$	$\bot\Leftarrow\bot$	$\neg$ **if** $\top$ **then** $\bot$ **else** $\bot$ **fi**
$\top\vee\top$	$\top=\top$	**if** $\bot$ **then** $\top$ **else** $\top$ **fi**
$\top\vee\bot$	$\neg(\top=\bot)$	$\neg$ **if** $\bot$ **then** $\top$ **else** $\bot$ **fi**
$\bot\vee\top$	$\neg(\bot=\top)$	**if** $\bot$ **then** $\bot$ **else** $\top$ **fi**
$\neg(\bot\vee\bot)$	$\bot=\bot$	$\neg$ **if** $\bot$ **then** $\bot$ **else** $\bot$ **fi**
$\top\Rightarrow\top$	$\neg(\top\ne\top)$	
$\neg(\top\Rightarrow\bot)$	$\top\ne\bot$	

Now I use the Completion and Instance Rules to pair axioms that differ in only one position. An axiom can participate in more than one pairing.

$\neg\neg\top$	$x\Rightarrow\top$	$\neg(\bot=\top)$
$\neg\bot$	$\neg(\top\Rightarrow\bot)$	$\neg(x\ne x)$
$\top\wedge\top$	$\bot\Rightarrow x$	$\top\ne\bot$
$\neg(x\wedge\bot)$	$\top\Leftarrow x$	$\bot\ne\top$
$\neg(\bot\wedge x)$	$\neg(\bot\Leftarrow\top)$	**if** $\top$ **then** $\top$ **else** x **fi**
$\top\vee x$	$x\Leftarrow\bot$	$\neg$ **if** $\top$ **then** $\bot$ **else** x **fi**
$x\vee\top$	$x=x$	**if** $\bot$ **then** x **else** $\top$ **fi**
$\neg(\bot\vee\bot)$	$\neg(\top=\bot)$	$\neg$ **if** $\bot$ **then** x **else** $\bot$ **fi**

It may seem that we can use symmetry to make the list even shorter. But the symmetry laws are proven from these axioms, so we can't.