4.0 Let $x$ be an integer variable. Let $P$ be a specification defined as follows.

$$P = x' = 0$$

Prove the refinement

$$P \iff \text{if } x > 0 \text{ then } x := x - 1. \ P$$
$$\quad \text{else if } x < 0 \text{ then } x := x + 1. \ P$$
$$\quad \text{else } ok \ fi \ fi$$

§ Using refinement by cases, I must prove three things:

$$x' = 0 \iff x > 0 \land (x := x - 1. \ x' = 0)$$
$$x' = 0 \iff x < 0 \land (x := x + 1. \ x' = 0)$$
$$x' = 0 \iff x = 0 \land ok$$

Let's start with the first.

$$x > 0 \land (x := x - 1. \ x' = 0)$$

use substitution law

$$= x > 0 \land x' = 0$$

specialization

$$\Rightarrow x' = 0$$

Now the middle one.

$$x < 0 \land (x := x + 1. \ x' = 0)$$

use substitution law

$$= x < 0 \land x' = 0$$

specialization

$$\Rightarrow x' = 0$$

And the last one.

$$x = 0 \land ok$$

replace $ok$

$$= x = 0 \land x' = x$$

transitivity

$$\Rightarrow x' = 0$$