398 (pure sets) A pure set is a set all of whose elements are pure sets. For example

$$\{\{null\}, \{\{null\}\}\}$$

First, $\{null\}$ is a pure set because all zero of its elements are pure sets. So $\{\{null\}\}$ is a pure set because its one element $\{null\}$ is a pure set. So $\{\{null\}, \{\{null\}\}\}$ is a pure set because both its elements are pure sets. All of mathematics can be implemented as pure sets. Define the bunch of texts representing pure sets.

After trying the question, scroll down to the solution.

§ I define *pureset* to be the bunch of all texts representing one pure set with the aid of *purebunch* , which is the bunch of all texts representing a nonempty bunch of pure sets.

$$pureset\text{::}\ \text{“}\{null\}\text{”},\ \text{“}\{\text{”};\ purebunch;\ \text{“}\}\text{”}$$

$$purebunch\text{::}\ pureset,\ purebunch;\ \text{“},\text{”};\ purebunch$$

These two construction axioms are mutually recursive. This makes the induction axiom a little more complicated.

$$P\text{::}\ \text{“}\{null\}\text{”},\ \text{“}\{\text{”};\ B;\ \text{“}\}\text{”}\ \land\ B\text{::}\ P,\ B;\ \text{“},\text{”};\ B\ \Rightarrow\ P\text{::}\ pureset\ \land\ B\text{::}\ purebunch$$

We could just as well write fixed-point construction and induction.

$$pureset\ =\ \text{“}\{null\}\text{”},\ \text{“}\{\text{”};\ purebunch;\ \text{“}\}\text{”}$$

$$purebunch\ =\ pureset,\ purebunch;\ \text{“},\text{”};\ purebunch$$

$$P = \text{“}\{null\}\text{”},\ \text{“}\{\text{”};\ B;\ \text{“}\}\text{”}\ \land\ B = P,\ B;\ \text{“},\text{”};\ B\ \Rightarrow\ P\text{::}\ pureset\ \land\ B\text{::}\ purebunch$$