A leafy tree is a tree with information residing only at the leaves. Design appropriate axioms for a binary leafy data-tree.

§

The following axioms constitute a strong theory of leafy trees.

\[
\begin{align*}
\text{leaf: } & X \rightarrow \text{tree} \\
\text{graft: } & \text{tree} \rightarrow \text{tree} \rightarrow \text{tree} \\
\text{leaf } X, \text{ graft } B B : B & \Rightarrow \text{ tree: } B \\
\text{graft } t u = \text{ leaf } x & \\
\text{leaf } x = \text{ leaf } y & \equiv x = y \\
\text{graft } t u = \text{ graft } v w & \equiv t = v \land u = w \\
\text{left } (\text{graft } t u) & = t \\
\text{right } (\text{graft } t u) & = u \\
\text{data } (\text{leaf } x) & = x
\end{align*}
\]

I have used a function \textit{leaf} to convert a data item to a one-item tree, and another function \textit{data} to retrieve it again. Another, simpler, approach is to consider that a data item is already a one-item tree. In that case, \textit{leaf} and \textit{data} aren't needed. The axioms are:

\[
\begin{align*}
\text{X: } & \text{tree} \\
\text{graft: } & \text{tree} \rightarrow \text{tree} \rightarrow \text{tree} \\
X, \text{ graft } B B : B & \Rightarrow \text{ tree: } B \\
\neg \text{ graft } t u : X & \\
\text{graft } t u = \text{ graft } v w & \equiv t = v \land u = w \\
\text{left } (\text{graft } t u) & = t \\
\text{right } (\text{graft } t u) & = u
\end{align*}
\]