A leafy tree is a tree with information residing only at the leaves. Design appropriate axioms for a binary leafy data-tree.

§ The following axioms constitute a strong theory of leafy trees.

\[
\begin{align*}
\text{leaf: } & \ X \to \text{tree} \\
\text{graft: } & \ \text{tree} \to \text{tree} \to \text{tree} \\
\text{leaf } X, \ \text{graft } B \ B: \ B & \Rightarrow \ \text{tree: } B \\
\text{graft } t \ u \neq \text{leaf } x & \\
\text{leaf } x = \text{leaf } y & \equiv \ x=y \\
\text{graft } t \ u = \text{graft } v \ w & \equiv \ t=v \land u=w \\
\text{left (graft } t \ u) & = t \\
\text{right (graft } t \ u) & = u \\
\text{data (leaf } x) & = x
\end{align*}
\]

I have used a function \( \text{leaf} \) to convert a data item to a one-item tree, and another function \( \text{data} \) to retrieve it again. Another, simpler, approach is to consider that a data item is already a one-item tree. In that case, \( \text{leaf} \) and \( \text{data} \) aren't needed. The axioms are:

\[
\begin{align*}
\text{X: } & \ \text{tree} \\
\text{graft: } & \ \text{tree} \to \text{tree} \to \text{tree} \\
\text{X, graft } B \ B: \ B & \Rightarrow \ \text{tree: } B \\
\neg \ \text{graft } t \ u: \ X & \ \\
\text{graft } t \ u = \text{graft } v \ w & \equiv \ t=v \land u=w \\
\text{left (graft } t \ u) & = t \\
\text{right (graft } t \ u) & = u
\end{align*}
\]