(general trees) In general, each node of a tree may have any number of subtrees.

(a) Design a data theory for general trees.

§

\[
\begin{align*}
& \text{tree} \neq \text{null} \\
& L: [\text{*tree}] \land x: X \Rightarrow \text{construct L} . x : \text{tree} \\
& L: [\text{*tree}] \land x: X \Rightarrow \text{root} (\text{construct L} . x) = x \\
& L: [\text{*tree}] \land x: X \land n < \#L \Rightarrow \text{child n} (\text{construct L} . x) = L n
\end{align*}
\]

If we also want an empty tree, we can add the axioms

emptree: tree

emptree \neq \text{construct L} . x

If we want an arity function, we add

\[
L: [\text{*tree}] \land x: X \Rightarrow \text{arity} (\text{construct L} . x) = \#L
\]

(b) Implement your theory.

(c) Prove your implementation.