39 Is there any harm in adding the axiom $0/0=5$ to Number Theory?

After trying the question, scroll down to the solution.

§ The number laws (Chapter 1) include five laws with $/$ in them. They are

(0) $x/1 = x$

(1) $-\infty<x<\infty \wedge x \neq 0 \Rightarrow 0/x = 0 \wedge x/x = 1$

(2) $z \neq 0 \Rightarrow x\times(y/z) = (x\times y)/z = x/(z/y)$

(3) $y \neq 0 \Rightarrow (x/y)/z = x/(y\times z)$

(4) $-\infty<x<\infty \Rightarrow x/\infty = 0 = x/-\infty$

Law (0) has denominator 1, so it doesn't apply to $0/0$. Law (1) applies with $x=0$, but it's a theorem because its antecedent is an antitheorem, so its consequent could be a theorem or an antitheorem or unclassified. Law (2) applies with $y=z=0$, but it's a theorem because its antecedent is an antitheorem, so its consequent could be a theorem or an antitheorem or unclassified. Law (3) applies with $x=z=0 \neq y$, but it just says $0/0 = 0/0$. Law (4) has denominators ∞ and $-\infty$, so it doesn't apply to $0/0$. So we can neither prove nor disprove $0/0 = 5$; it's neither a theorem nor an antitheorem. We are free to add it as an axiom or as an antiaxiom. But it would be most inelegant to do so. Why 5? We could equally well choose $0/0 = x$ for any extended real x.

The bunch laws (Chapter 2) include two more laws with $/$ in them. They are

(5) $\infty, -\infty\colon\ x/0$

(6) $xreal\colon 0/0$

Law (6) says that all extended real numbers are included in $0/0$. So with these bunch laws, we cannot add the axiom $0/0 = 5$ to Number Theory.