In Subsection 7.1.3 we designed a program-stack theory so weak that we could add axioms to count pushes and pops without inconsistency. Design a similarly weak data-stack theory.

$\quad stack \neq null$

$\quad \text{push stack } X: \quad stack$

$\quad \text{top (push } s \ x) = x$

$\quad \text{top (balance } s) = \text{top } s$

$\quad s, \text{pop (balance (push } s \ X)): \quad balance \ s$

Now we can add

$\quad \text{count: stack } \rightarrow \text{nat}$

$\quad \text{count (push } s \ x) = \text{count } s + 1$

$\quad \text{count (pop } s) = \text{count } s + 1$

We don't need an empty stack to start; we can just take note of the count at the start and subtract that whenever we want the relative count. Here's an implementation.

$\quad stack = [\text{nat;} \ *X]$

$\quad push = \langle s: \text{stack} \rightarrow \langle x: X \rightarrow [s \ 0 + 1]; s[1..#s]; [x] \rangle \rangle$

$\quad pop = \langle s: \text{stack} \rightarrow [s \ 0 + 1]; s[1..#s-1] \rangle$

$\quad top = \langle s: \text{stack} \rightarrow s[#s-1] \rangle$

$\quad count = \langle s: \text{stack} \rightarrow s \ 0 \rangle$

To prove the implementation, we need to define $balance$

$\quad balance = \langle s: \text{stack} \rightarrow §t: \text{stack} \cdot s[1..#s]=t[1..#t] \rangle$

but we don't need to implement it.