387 Bunch $this$ is defined by the construction and induction axioms

$$2, 2{\times}this\colon\ this$$

$$2, 2{\times}B\colon\ B \quad\Rightarrow\quad this\colon B$$

Bunch $that$ is defined by the construction and induction axioms

$$2, that{\times}that\colon\ that$$

$$2, B{\times}B\colon\ B \quad\Rightarrow\quad that\colon B$$

Prove $this = that$.

After trying the question, scroll down to the solution.

§ Recursive construction for *this* produces

$$this_n = 2^{(0,..n)+1}$$

So $this_\infty = 2^{nat+1}$ and it satisfies both construction and induction for *this* .
Hence $this = 2^{nat+1}$.
Recursive construction for *that* produces

$that_n$ = 2 to the power $(0,..2^n)+1$ (I can't do 4 levels typographically)

So $that_\infty$ = 2 to the power $(0,..2^\infty)+1$. I just need $2^\infty=\infty$ to say $that_\infty = 2^{nat+1}$. But even without it, I can check whether 2^{nat+1} satisfies both construction and induction for *that* , and I find that it does, so $that = 2^{nat+1}$.
Therefore $this=that$.