What elements can be proven in \( P \) from the axiom \( P = 1, x, -P, P + P, P \times P \)? Prove \( 2x^2 - 1: P \)

After trying the question, scroll down to the solution.
All polynomials in one variable $x$ (so really that's monomials) with integer coefficients are in $P$.

1. $P$ and $P + P : P$ therefore 1 + 1 : $P$ therefore 2 : $P$.
2. $x : P$ and $P \times P : P$ therefore $x \times x : P$ therefore $x^2 : P$.
3. $2 : P$ and $x^2 : P$ and $P \times P : P$ therefore $2 \times x^2 : P$.
4. $2 \times x^2 : P$ and $- P : P$ therefore $-2 \times x^2 : P$.
5. $-2 \times x^2 : P$ and $1 : P$ and $P + P : P$ therefore $-2 \times x^2 + 1 : P$.
6. $-2 \times x^2 + 1 : P$ and $- P : P$ therefore $- (2 \times x^2 + 1) : P$ therefore $2 \times x^2 - 1 : P$. 
