Let $x$ be an integer variable.

(a) Using the recursive time measure, add time and then find the strongest implementable specification $S$ that you can find for which

\[
S \iff \begin{cases}
    \text{if } x=0 \text{ then } ok \\
    \text{else if } x>0 \text{ then } x:= x-1. \ S \\
    \text{else } x' \geq 0. \ S \fi \fi
\]

Assume that $x' \geq 0$ takes no time.

Adding time,

\[
S \iff \begin{cases}
    \text{if } x=0 \text{ then } ok \\
    \text{else if } x>0 \text{ then } x:= x-1, \ t:= t+1. \ S \\
    \text{else } x' \geq 0 \land t'=t. \ t:= t+1. \ S \fi \fi
\]

the strongest implementable solution for $S$ is

\[
x' = 0 \land \text{if } x \geq 0 \text{ then } t' = t+x \text{ else } t' \geq t+1 \fi
\]

If we replace $x' \geq 0 \land t'=t$ by $x:= c$ where $c$ is an arbitrary natural number, then we can prove

\[
x' = 0 \land \text{if } x \geq 0 \text{ then } t' = t+x \text{ else } t' = t+1+c \fi
\]

(b) What do we get from recursive construction starting with $t' \geq t$?

\[
S_n = \begin{cases}
    0 \leq x < n \land x' = 0 \land t' = t+x \\
    \lor \ 0 \leq x < n \land t' \geq t+n \\
    \lor \ x < 0 \land x' = 0 \land t+1 \leq t' < t+n
\end{cases}
\]

\[
S_\infty = \begin{cases}
    0 \leq x \land x' = 0 \land t' = t+x \\
    \lor \ x < 0 \land t' = \infty \\
    \lor \ x < 0 \land x' = 0 \land t+1 \leq t' < \infty
\end{cases}
\]

$S_\infty$ is a solution to the given implication, but not as strong as the solution shown in part (a). It is interesting to note that if the given implication were an equation, then $S_\infty$ would not be a solution (fixed-point), but the solution of part (a) would still be a solution. $\text{LIM } n \cdot S_n$ is the same as $S_\infty$. 
