Here is a possible alternative construction axiom for \( \text{nat} \).

\[ 0, 1, \text{nat+nat: nat} \]

(a) What induction axiom goes with it?
(b) Are the construction axiom given and your induction axiom of part (a) satisfactory as a definition of \( \text{nat} \)?

After trying the question, scroll down to the solution.
(a) What induction axiom goes with it?
§ 0, 1, B+B: B ⇒ nat: B

(b) Are the construction axiom given and your induction axiom of part (a) satisfactory as a definition of nat?
§ Yes. To prove they are sufficient to define nat, use them to prove ordinary nat construction and induction. So, assume the new construction and induction (and do not assume anything else about nat). Now prove ordinary nat construction.

\[
0, \text{nat}+1: \text{nat} \quad \text{UNFINISHED}
\]

Now prove ordinary nat induction.

\[
0, B+1: B \Rightarrow \text{nat}: B \quad \text{UNFINISHED}
\]

We don't really need to prove they are necessary to define nat, but if we want to, assume ordinary nat construction and induction (and do not assume anything else about nat). Now prove the new construction.

\[
0, 1, \text{nat}+\text{nat}: \text{nat} \quad \text{from ordinary construction, we have both 0: nat and 1: nat}
\]

\[
\equiv \quad \text{nat}+\text{nat}: \text{nat} \quad \text{bunch-element conversion law}
\]

\[
\equiv \quad \forall n: \text{nat}+\text{nat} \exists m: \text{nat} \cdot n=m \quad \text{UNFINISHED}
\]

\[
\equiv \quad \top
\]

Now prove the new induction.

\[
0, 1, B+B: B \Rightarrow \text{nat}: B \quad \text{UNFINISHED}
\]

\[
\equiv \quad \top
\]