Section 6.1 defines program \textit{zap} by the fixed-point equation

\[
\text{\textit{zap} } \equiv \text{ if } x=0 \text{ then } y:= 0 \text{ else } x:= x-1. \ t:= t+1. \ \text{\textit{zap} fi}
\]

(a) Prove \textit{zap} \implies x \geq 0 \implies x' = y' = 0 \land t' = t+x.

§ Maybe induction on \( x \)?

(b) Prove \( x \geq 0 \land x' = y' = 0 \land t' = t+x \implies \text{\textit{zap}}. \)

(c) What axiom is needed to make \textit{zap} the weakest fixed-point?

§ \( (\forall \sigma, \sigma' \cdot P = \text{ if } x=0 \text{ then } y:= 0 \text{ else } x:= x-1. \ t:= t+1. \ P \text{ fi} ) \implies (\forall \sigma, \sigma' \cdot \text{\textit{zap} \iff P}) \)

(d) What axiom is needed to make \textit{zap} the strongest fixed-point?

§ \( (\forall \sigma, \sigma' \cdot P = \text{ if } x=0 \text{ then } y:= 0 \text{ else } x:= x-1. \ t:= t+1. \ P \text{ fi} ) \implies (\forall \sigma, \sigma' \cdot P \iff \text{\textit{zap}}) \)

(e) Section 6.1 gives six solutions to this equation. Find more solutions. Hint: strange things can happen at time \( \infty \).