Let \( \emptyset \) and \( \mathbf{1} \) be two new values. Define a new empty sequence \( \texttt{nil} \) and a new join operator \( ; \) (precedence 5) that makes sequences of \( \emptyset \)s and \( \mathbf{1} \)s, and define a new addition operator \( + \) (precedence 4) on those sequences as follows.

\[
\begin{align*}
\texttt{nil}; a & = a = a; \texttt{nil} \\
(a; b), c & = a; (b; c) \\
b; \emptyset & = b \\
(b; \emptyset) + \mathbf{1} & = b; \mathbf{1} \\
(b; \mathbf{1}) + \mathbf{1} & = (b+\mathbf{1}); \emptyset \\
a+b = b+a \\
(a+b)+c & = a+(b+c)
\end{align*}
\]

Define the binary natural numbers \( \textit{binat} \) as follows.

\[
\begin{align*}
\emptyset, \textit{binat}+1 & : \textit{binat} \\
\emptyset, \textit{B}+1 : B & \Rightarrow \textit{binat}: B
\end{align*}
\]

For \( n : 0, \mathbf{1} \) and \( b : \textit{binat} \) prove

(a) \( b+b = (b; \emptyset) \)

(b) \( (b; n) = b+b+n \)