365 We propose to define a new programming connective $P \blacklozenge Q$. What properties of $\blacklozenge$ are essential? Why?

After trying the question, scroll down to the solution.

§ It must be defined for all specifications P and Q, not just for programs, so that it can be used during program development. It must be implementable, which means

$$(\forall\sigma\cdot\ \exists\sigma'\cdot\ P \land t'\geq t) \land (\forall\sigma\cdot\ \exists\sigma'\cdot\ Q \land t'\geq t) \Rightarrow (\forall\sigma\cdot\ \exists\sigma'\cdot\ (P \blacklozenge Q) \land t'\geq t)$$

(This property can be contested because **ensure** is not implementable.) It must be monotonic in both operands so that Refinement by Steps and Refinement by Parts can be used.

If $A \Leftarrow B \blacklozenge C$ and $B \Leftarrow D$ and $C \Leftarrow E$ are theorems,
then $A \Leftarrow D \blacklozenge E$ is a theorem.
If $A \Leftarrow B \blacklozenge C$ and $D \Leftarrow E \blacklozenge F$ are theorems,
then $A\land D \Leftarrow B\land E \blacklozenge C\land F$ is a theorem.

(Since $\blacklozenge$ is a symmetric symbol, perhaps it ought to be a symmetric operator

$$P \blacklozenge Q = Q \blacklozenge P$$

but that's not an essential point and there are lots of counterexamples.)