356 (Bob asks Alice for a date) Bob wants to ask Alice out on a date. He knows his success rate (the fraction of all the women he's asked out so far who have said yes). And he knows Alice's agreement rate (the fraction of all the guys who have asked Alice out so far that she has said yes to). What is the probability that Alice will say yes to Bob if

(a) Bob's success rate is $2/3$ and Alice's agreement rate is $1/2$?

(b) Bob's success rate is $2/3$ and Alice's agreement rate is $2/3$?

After trying the question, scroll down to the solution.

(a) Bob's success rate is $2/3$ and Alice's agreement rate is $1/2$?

§ Let a be the answer yes (1) or no (0) that Alice will give. Bob's success rate tells us

$$
\begin{aligned}
& \textbf{if } 2/3 \textbf{ then } a:=1 \textbf{ else } a:=0 \textbf{ fi} \\
= \;& 2/3 \times (a'=1) \;+\; 1/3 \times (a'=0) \\
= \;& (a'+1)/3
\end{aligned}
$$

Alice's agreement rate tells us

$$
\begin{aligned}
& \textbf{if } 1/2 \textbf{ then } a:=1 \textbf{ else } a:=0 \textbf{ fi} \\
= \;& 1/2 \times (a'=1) \;+\; 1/2 \times (a'=0) \\
= \;& 1/2
\end{aligned}
$$

We know both these distributions, so we multiply them together. But multiplying distributions does not necessarily result in a distribution, so we must normalize the result (divide by the sum) to obtain a distribution.

$$
\begin{aligned}
& ((a'+1)/3 \times 1/2) \;/\; (\Sigma a' \cdot (a'+1)/3 \times 1/2) \\
= \;& ((a'+1)/6) \;/\; ((0+1)/3 \times 1/2 \;+\; (1+1)/3 \times 1/2) \\
= \;& ((a'+1)/6) \;/\; (1/2) \\
= \;& (a'+1)/3
\end{aligned}
$$

which says Alice agrees with probability $2/3$, and declines with probability $1/3$. This is exactly Bob's success rate. That's because Alice's agreement rate $1/2$ is the identity for parallel composition in a 2-state space. A uniform distribution gives us no information about what will happen. Alice's record of agreement gives Bob no idea whether she will say yes or no to him.

(b) Bob's success rate is $2/3$ and Alice's agreement rate is $2/3$?

§ Bob's success rate tells us $(a'+1)/3$ as in part (a), and Alice's agreement rate now tells us $(a'+1)/3$ also. We know both these distributions, so we multiply them together. But multiplying distributions does not necessarily result in a distribution, so we must normalize the result (divide by the sum) to obtain a distribution.

$$
\begin{aligned}
& ((a'+1)/3 \times (a'+1)/3) \;/\; (\Sigma a' \cdot (a'+1)/3 \times (a'+1)/3) \\
= \;& ((a'+1)^2 / 9) \;/\; (1/3 \times 1/3 \;+\; 2/3 \times 2/3) \\
= \;& ((a'+1)^2 / 9) \;/\; (1/9 \;+\; 4/9) \\
= \;& ((a'+1)^2 / 9) \;/\; (5/9) \\
= \;& (a'+1)^2 / 5
\end{aligned}
$$

which says yes with probability $4/5$ and no with probability $1/5$. Bob's and Alice's distributions are both biased toward yes, and in parallel they produce a distribution even more biased toward yes.

If a probability distribution were an objective attribute of a situation, independent of anyone's knowledge of the situation, we might suppose that Bob is trying to determine this probability distribution. The first fact Bob considers suggests that the distribution is **if** 2/3 **then** a:= 1 **else** a:= 0 **fi** . If the next fact Bob learns also suggests that the distribution is **if** 2/3 **then** a:= 1 **else** a:= 0 **fi** , this would strengthen, or increase, his belief that the distribution really is **if** 2/3 **then** a:= 1 **else** a:= 0 **fi** . But Bob is not trying to discover a distribution. He is trying to discover whether Alice will say yes. A probability distribution is a way of expressing whether Alice will say yes. The first fact suggests (with weight $2/3$) that she will. When the second fact also suggests (with weight $2/3$) that she will, this strengthens, or increases, Bob's belief that she will say yes (to weight $4/5$).