349 (two coins) We flip a pair of coins repeatedly. We ignore the flip when both coins show tail, and we count the flip when at least one coin shows head. We continue flipping until we have counted 100 flips. Of these 100 flips, what fraction show two heads?

After trying the question, scroll down to the solution.

§ Let the coins be binary variables a and b. Let head be 1 and tail be 0. Let c be a natural variable counting the number of times there is at least one head. Let d be a natural variable counting the number of times there are both a head and a tail. Here is the program followed by the question.

$c:= 0$. $d:= 0$.
while $c<100$ **do if** 1/2 **then** $a:= 0$ **else** $a:= 1$ **fi**.
if 1/2 **then** $b:= 0$ **else** $b:= 1$ **fi**.
if $a \lor b$ **then** $c:= c+1$. **if** $a \land b$ **then** $d:= d+1$ **else** *ok* **fi**
else *ok* **fi od**.
$d/100$

If we find the right loop hypothesis and do the calculation, we will get the right answer. But there is an easier way. A flip makes at least one coin be a head with probability

$$(a' \lor b') \,/\, (\Sigma a', b' \cdot a' \lor b') \qquad \text{do the sum}$$
$$= \; (a' \lor b') \,/\, 3$$

Now we follow that by the question whether both coins are head.

$$(a' \lor b') \,/\, 3.\; a \land b \qquad \text{replace } .$$
$$= \; \Sigma a'', b'' \cdot (a'' \lor b'') \,/\, 3 \times (a'' \land b'') \qquad \text{do the sum}$$
$$= \; 1/3$$