347 (one coin) Repeatedly flip a coin until you get a head. Prove that it takes n flips with probability 2^{-n}. With an appropriate definition of R, the program is

$R \Leftarrow t := t+1.$ **if** *rand* 2 **then** *ok* **else** R **fi**

After trying the question, scroll down to the solution.

§ Starting with the right side, using $1/2$ for *rand* 2 and $(t'>t) \times 2^{t-t'}$ for R :

$t := t+1$. **if** $1/2$ **then** $t'=t$ **else** $(t'>t) \times 2^{t-t'}$ **fi** substitution law

$=$ **if** $1/2$ **then** $t'=t+1$ **else** $(t'>t+1) \times 2^{t+1-t'}$ **fi** replace **if**

$=$ $(t'=t+1) / 2 \; + \; (t'>t+1) \times 2^{t+1-t'} / 2$

$=$ $(t'=t+1) \times 2^{t-t'} \; + \; (t'>t+1) \times 2^{t-t'}$

$=$ $(t'>t) \times 2^{t-t'}$

$=$ R