

339 (Boole's binaries) If $\top=1$ and $\perp=0$, express

- (a) $\neg a$
- (b) $a \wedge b$
- (c) $a \vee b$
- (d) $a \Rightarrow b$
- (e) $a \Leftarrow b$
- (f) $a = b$
- (g) $a \neq b$

using only the following symbols (in any quantity)

- (i) $0 1 a b () + - \times$
- (ii) $0 1 a b () - \uparrow \downarrow$

That's $7 \times 2 = 14$ questions.

After trying the question, scroll down to the solution.

§		(i)	(ii)
(a)	$\neg a$	$= 1-a$	$= 1-a$
(b)	$a \wedge b$	$= a \times b$	$= a \downarrow b$
(c)	$a \vee b$	$= a + b - a \times b$	$= a \uparrow b$
(d)	$a \Rightarrow b$	$= 1 - a + a \times b$	$= (1-a) \uparrow b$
(e)	$a \Leftarrow b$	$= 1 - b + a \times b$	$= a \uparrow (1-b)$
(f)	$a = b$	$= 1 - a - b + 2 \times a \times b$	$= (a \uparrow (1-b)) \downarrow ((1-a) \uparrow b)$
(g)	$a \neq b$	$= a + b - 2 \times a \times b$	$= (a \downarrow (1-b)) \uparrow ((1-a) \downarrow b)$