Here is a possible alternative construction axiom for $\text{nat}$.

$$0, 1, \text{nat}+\text{nat}: \text{nat}$$

(a) What induction axiom goes with it?

§ $$0, 1, \text{B}+\text{B}: \text{B} \Rightarrow \text{nat}: \text{B}$$

(b) Are the construction axiom given and your induction axiom of part (a) satisfactory as a definition if $\text{nat}$?

§ Yes. To prove they are sufficient to define $\text{nat}$, use them to prove ordinary $\text{nat}$ construction and induction. So, assume the new construction and induction (and do not assume anything else about $\text{nat}$). Now prove ordinary $\text{nat}$ construction.

$$0, \text{nat}+1: \text{nat}$$

= $T$

Now prove ordinary $\text{nat}$ induction.

$$0, \text{B}+1: \text{B} \Rightarrow \text{nat}: \text{B}$$

= $T$

We don't really need to prove they are necessary to define $\text{nat}$, but if we want to, assume ordinary $\text{nat}$ construction and induction (and do not assume anything else about $\text{nat}$). Now prove the new construction.

$$0, 1, \text{nat}+\text{nat}: \text{nat}$$

from ordinary construction, we have both $0: \text{nat}$ and $1: \text{nat}$

= $\text{nat}+\text{nat}: \text{nat}$

= $\forall n: \text{nat}+\text{nat} \cdot \exists m: \text{nat} \cdot n=m$

= $T$

Now prove the new induction.

$$0, 1, \text{B}+\text{B}: \text{B} \Rightarrow \text{nat}: \text{B}$$

= $T$