Here is a possible alternative construction axiom for \( \text{nat} \).
\[
0, 1, \text{nat} + \text{nat} : \text{nat}
\]

(a) What induction axiom goes with it?
\[
\begin{align*}
0, 1, B + B : B & \Rightarrow \text{nat} : B \\
\end{align*}
\]

(b) Are the construction axiom given and your induction axiom of part (a) satisfactory as a definition of \( \text{nat} \)?
\[
\begin{align*}
\text{§} & \quad \text{Yes. To prove they are sufficient to define \( \text{nat} \), use them to prove ordinary \( \text{nat} \) construction and induction. So, assume the new construction and induction (and do not assume anything else about \( \text{nat} \)). Now prove ordinary \( \text{nat} \) construction.} \\
& \quad \text{0, nat+1: nat} \\
& \quad = T \\
& \text{Now prove ordinary \( \text{nat} \) induction.} \\
& \quad 0, B+1 : B \Rightarrow \text{nat} : B \\
& \quad = T
\end{align*}
\]

We don't really need to prove they are necessary to define \( \text{nat} \), but if we want to, assume ordinary \( \text{nat} \) construction and induction (and do not assume anything else about \( \text{nat} \)). Now prove the new construction.
\[
\begin{align*}
0, 1, \text{nat} + \text{nat} : \text{nat} & \quad \text{from ordinary construction, we have both 0: nat and 1: nat} \\
& \quad \text{nat} + \text{nat} : \text{nat} \\
& \quad = \forall n : \text{nat}, \exists m : \text{nat} \cdot n = m \\
& \quad = T
\end{align*}
\]

Now prove the new induction.
\[
\begin{align*}
0, 1, B + B : B & \Rightarrow \text{nat} : B \\
& \quad = T
\end{align*}
\]