Here is a procedure applied to an argument.

\[ \langle x: \text{int} \rightarrow a:=x. \ b:=x \rangle (a+1) \]

Suppose, by mistake, we replace both occurrences of \( x \) in the body with the argument. What do we get? What should we get? (This mistake is known as “call-by-name”.)

\[ \langle x: \text{int} \rightarrow a:=x. \ b:=x \rangle (a+1) \]

as directed, replace both occurrences of \( x \)

\[ \equiv a:=a+1. \ b:=a+1 \]

\[ \equiv a' = a+1 \land b' = a+2 \]

On page 39, Exercise 110(k) says that it is a mistake to replace the \( x \) after the composition. Here's what we should get.

\[ \langle x: \text{int} \rightarrow a:=x. \ b:=x \rangle (a+1) \]

expand the two assignments

\[ \equiv \langle x: \text{int} \rightarrow a'=x \land b'=b. \ a'=a \land b'=x \rangle (a+1) \]

definition of sequential composition

\[ \equiv \langle x: \text{int} \rightarrow \exists a'', b''. \ a''=x \land b''=b \land a'=a'' \land b'=x \rangle (a+1) \]

apply

\[ \equiv a'=b'=a+1 \]

OR

\[ \langle x: \text{int} \rightarrow a:=x. \ b:=x \rangle (a+1) \]

expand the last assignment

\[ \equiv \langle x: \text{int} \rightarrow a'=a \land b'=x \rangle (a+1) \]

substitution law

\[ \equiv \langle x: \text{int} \rightarrow a'=x \land b'=x \rangle (a+1) \]

apply

\[ \equiv a'=b'=a+1 \]