337 Here is a procedure applied to an argument.

$$\langle x{:}\ int\cdot\ a{:}=x.\ \ b{:}=x\rangle\,(a+1)$$

Suppose, by mistake, we replace both occurrences of x in the body with the argument. What do we get? What should we get? (This mistake is known as "call-by-name".)

After trying the question, scroll down to the solution.

§ $\langle x\text{: } int\cdot\ \ a\text{:= } x.\ \ b\text{:= } x\rangle\ (a+1)$ as directed, replace both occurrences of x

$= \quad a\text{:= } a+1.\ \ b\text{:= } a+1$

$= \quad a' = a+1 \ \ \wedge\ \ b' = a+2$

On page 39, Exercise 110(k) says that it is a mistake to replace the x after the composition. Here's what we should get.

$\langle x\text{: } int\cdot\ \ a\text{:= } x.\ \ b\text{:= } x\rangle\ (a+1)$ expand the two assignments

$= \quad \langle x\text{: } int\cdot\ \ a'=x \wedge b'=b.\ \ a'=a \wedge b'=x\rangle\ (a+1)$ definition of sequential composition

$= \quad \langle x\text{: } int\cdot\ \ \exists a'', b''\cdot\ \ a''=x \wedge b''=b \wedge a'=a'' \wedge b'=x\rangle\ (a+1)$

$= \quad \langle x\text{: } int\cdot\ \ a'=b'=x\rangle\ (a+1)$ apply

$= \quad a'=b'=a+1$

OR

$\langle x\text{: } int\cdot\ \ a\text{:= } x.\ \ b\text{:= } x\rangle\ (a+1)$ expand the last assignment

$= \quad \langle x\text{: } int\cdot\ \ a\text{:= } x.\ \ a'=a \wedge b'=x\rangle\ (a+1)$ substitution law

$= \quad \langle x\text{: } int\cdot\ \ a'=x \wedge b'=x\rangle\ (a+1)$

$= \quad \langle x\text{: } int\cdot\ \ a'=b'=x\rangle\ (a+1)$ apply

$= \quad a'=b'=a+1$