332 The specification **wait** w where w is a length of time, not an instant of time, describes a delay in execution of time w. Formalize and implement it using

(a) the recursive time measure.

(b) the real time measure (assume any positive operation times you need).

After trying the question, scroll down to the solution.

(a)§ If t is an extended natural variable and w is an extended natural expression, then define

$\textbf{wait } w \ = \ t{:=}\ t{+}w$

and refine it this way:

$\textbf{wait } w \ \Leftarrow \ \textbf{frame } t\cdot\ \textbf{new } c{:}\ xnat := w\cdot\ \ t' = t{+}c$

$t' = t{+}c \ \Leftarrow \ \textbf{if } c{=}0 \textbf{ then } ok \textbf{ else } c{:=}\ c{-}1.\ \ t{:=}\ t{+}1.\ \ t' = t{+}c \textbf{ fi}$

Proof of first refinement:

$$\begin{array}{lll} & \textbf{frame } t\cdot\ \textbf{new } c{:}\ xnat := w\cdot\ \ t' = t{+}c & \\ = & \textbf{frame } t\cdot\ \exists c{:}\ w\cdot\ \exists c'{:}\ xnat\cdot\ \ t' = t{+}c & c' \text{ is unused; } \exists \text{ law} \\ = & \textbf{frame } t\cdot\ \ t' = t{+}w & \textbf{frame} \text{ law} \\ = & t{:=}\ t{+}w & \\ = & \textbf{wait } w & \end{array}$$

Proof of last refinement, first case, assuming the nonlocal variables are x :

$$\begin{array}{lll} & c{=}0 \wedge ok & \text{expand } ok \\ = & c{=}0 \wedge x'{=}x \wedge c'{=}c \wedge t'{=}t & \text{context and specialization} \\ \Rightarrow & t' = t{+}c & \end{array}$$

Proof of last refinement, last case:

$$\begin{array}{lll} & c{>}0 \wedge (c{:=}\ c{-}1.\ \ t{:=}\ t{+}1.\ \ t' = t{+}c) & \text{substitution law twice; specialization} \\ \Rightarrow & t' = t{+}c & \end{array}$$

(b)§ This time t is a nonnegative extended real variable and w is a nonnegative extended real expression. The solution can be like part (a), but in the real time measure, we have to account for the time to make the test (which was $c{=}0$ in part (a)) and to make a conditional branch, and the time for the assignment (which was $c{:=}\ c{-}1$ in part (a)), and the time for the recursive call. I'll use time 1 for all three. As in part (a), we can introduce a counter c initialized to w and count down. But w here is real, not necessarily an integer, so either the test must be $c{\le}0$, or the initial value of c must be rounded up. I'll do the latter. Define

$\textbf{wait } w \ = \ t{:=}\ t + 3{\times}(ceil\ w) + 1$

and refine it this way:

$\textbf{wait } w \ \Leftarrow \ \textbf{frame } t\cdot\ \textbf{new } c{:}\ xnat := ceil\ w\cdot\ \ t' = t + 3{\times}c + 1$

$t' = t + 3{\times}c + 1 \ \Leftarrow$

$t{:=}\ t{+}1.\ \ \textbf{if } c{=}0 \textbf{ then } ok \textbf{ else } t{:=}\ t{+}1.\ \ c{:=}\ c{-}1.\ \ t{:=}\ t{+}1.\ \ t' = t + 3{\times}c + 1 \textbf{ fi}$

Proof of first refinement:

$$\begin{array}{lll} & \textbf{frame } t\cdot\ \textbf{new } c{:}\ xnat := ceil\ w\cdot\ \ t' = t + 3{\times}c + 1 & \\ = & \textbf{frame } t\cdot\ \exists c{:}\ ceil\ w\cdot\ \exists c'{:}\ xnat\cdot\ \ t' = t + 3{\times}c + 1 & c' \text{ is unused; } \exists \text{ law} \\ = & \textbf{frame } t\cdot\ \ t' = t + 3{\times}(ceil\ w) + 1 & \textbf{frame} \text{ law} \\ = & t{:=}\ t + 3{\times}(ceil\ w) + 1 & \\ = & \textbf{wait } w & \end{array}$$

Proof of last refinement, assuming the nonlocal variables are x :

$$\begin{array}{lll} & t{:=}\ t{+}1.\ \ \textbf{if } c{=}0 \textbf{ then } ok \textbf{ else } t{:=}\ t{+}1.\ \ c{:=}\ c{-}1.\ \ t{:=}\ t{+}1.\ \ t' = t + 3{\times}c + 1 \textbf{ fi} & \\ & & \text{substitution law 3 times} \\ = & t{:=}\ t{+}1.\ \ \textbf{if } c{=}0 \textbf{ then } ok \textbf{ else } t' = t + 3{\times}c \textbf{ fi} & \text{expand } ok \\ = & t{:=}\ t{+}1.\ \ \textbf{if } c{=}0 \textbf{ then } c'{=}c \wedge x'{=}x \wedge t'{=}t \textbf{ else } t' = t + 3{\times}c \textbf{ fi} & \text{substitution law} \\ = & \textbf{if } c{=}0 \textbf{ then } c'{=}c \wedge x'{=}x \wedge t'{=}t{+}1 \textbf{ else } t' = t + 3{\times}c + 1 \textbf{ fi} & \text{use context } c{=}0 \\ = & \textbf{if } c{=}0 \textbf{ then } c'{=}c \wedge x'{=}x \wedge t' = t + 3{\times}c + 1 \textbf{ else } t' = t + 3{\times}c + 1 \textbf{ fi} & \text{specialize} \\ \Rightarrow & t' = t + 3{\times}c + 1 & \end{array}$$