This is a hard question. Let $N x i = \exists j: 0,..: L j = x$. So $N x i$ is the number of occurrences of item $x$ in list $L$ before index $i$. The problem can be stated formally as

$$\forall x:: N x (#L) > #L/2 \Rightarrow m' = x$$

or

$$\forall x:: x+ m' \Rightarrow N x (#L) \leq #L/2$$

The invariant we need for the for-loop is defined as

$$A i = (i \leq 2xe \wedge N m i \leq e \wedge \forall x:: x + m \Rightarrow N x i \leq i - e)$$

We must prove two refinements:

$$\forall x:: x + m' \Rightarrow N x (#L) \leq #L/2 \iff \text{var e:: nart := 0, A 0 \Rightarrow A'(#L)}$$

$$i: 0,..#L \wedge A i \Rightarrow A'(i+1) \iff$$

$$\text{if } m = L i \text{ then } e := e+1$$

$$\text{else if } i = 2xe \text{ then } m := L i. \ e := e+1$$

$$\text{else ok fi od}$$

The first refinement is proven as follows:

$$\forall x:: x + m' \Rightarrow N x (#L) \leq #L/2$$

The second refinement to be proven can be broken into three cases. The first case is

$$(i: 0,..#L \wedge A i \Rightarrow A'(i+1) \iff m = L i \wedge (e := e+1))$$

The second case is

$$(i: 0,..#L \wedge A i \Rightarrow A'(i+1) \iff m + Li \wedge i = 2xe \wedge (m := L i. \ e := e+1))$$

simplify antecedent, and use its equations

$$2xe + 1 \leq 2(e+1) \wedge N(Li)(i+1) \leq e+1 \wedge (\forall x:: x \neq L i \Rightarrow N x (i+1) \leq e)$$
The third case is

\[(i: 0..\#L \land A i \Rightarrow A'(i+1) \iff m \not\in L i \land i+2\times e \land ok) \text{ portation}\]

\[\equiv m \not\in L i \land i+2\times e \land ok \land i: 0..\#L \land A i \Rightarrow A'(i+1)\]

\[\equiv m \not\in L i \land i+2\times e \land e'=e \land m'=m \land i: 0..\#L\]

\[\land i \leq 2\times e \land N m i \leq e \land (\forall x: x \not\in L \Rightarrow N x i \leq i-e)\]

\[\Rightarrow i+1 \leq 2\times e' \land N m'(i+1) \leq e' \land (\forall x: x \not\in L \Rightarrow N x (i+1) \leq i+1-e')\]

in the antecedent we have \(m' = m \not\in L i\), so \(N m'(i+1) = N m i\).

And for all \(x, N x (i+1): N x i, N x i + 1\).

\[\equiv \top\]

Note that the program is correct even though the initial value of variable \(m\) is arbitrary.

(b) 
\[
\texttt{var s: nat := 0;} \\
\texttt{for i:= 0..\#L} \\
\texttt{do if m = L i then ok} \\
\texttt{else if i = 2\times s then m:= L i} \\
\texttt{else s:= s+1 fi od} \\
\]

\[\$\text{This is the same as part (a) with } s = i-e.\]