Formally, we want the result

$$\text{if } 1/2 \text{ then } x' = \text{head} \text{ else } x' = \text{tail} \text{ fi}$$

The procedure apparently achieves slightly more:

$$\text{if } 1/2 \text{ then } x' = \text{head} \land y' = \text{tail} \text{ else } x' = \text{tail} \land y' = \text{head} \text{ fi}$$

which can be simplified to

$$(x' \neq y') / 2$$

If we call that result $R$, then one understanding of the procedure is the program

$$R = \begin{cases} \text{if } p \text{ then } x := \text{head} \text{ else } x := \text{tail} \text{ fi} \\ \text{if } p \text{ then } y := \text{head} \text{ else } y := \text{tail} \text{ fi} \\ \text{if } x = y \text{ then } R \text{ else } \text{ok fi} \end{cases}$$

Another understanding of the procedure is the program

$$R = \begin{cases} \text{if } p \text{ then } x := \text{head} \text{ else } x := \text{tail} \text{ fi} \\ S \end{cases}$$

$$S = \begin{cases} \text{if } p \text{ then } y := \text{head} \text{ else } y := \text{tail} \text{ fi} \\ \text{if } x = y \text{ then } x := y \text{; } S \text{ else } \text{ok fi} \end{cases}$$

The informal description could reasonably be understood either way; it is ambiguous. If two people with different understandings of the informal description of the procedure ask each other whether it is clear and understood, they will each say yes, and a long argument about whether the procedure produces the desired result will ensue. In contrast to that, the programs are unambiguous. With them we don't need to argue; we just calculate. Let me begin with the first one.

$$\begin{cases} \text{if } p \text{ then } x := \text{head} \text{ else } x := \text{tail} \text{ fi} \\ \text{if } p \text{ then } y := \text{head} \text{ else } y := \text{tail} \text{ fi} \\ \text{if } x = y \text{ then } R \text{ else } \text{ok fi} \end{cases}$$

$$= \Sigma x'', y'' \cdot (p x' = \text{head}) + (1-p) x' = \text{tail} \times (p y' = \text{head}) + (1-p) y' = \text{tail} \times ((x'' = y'') \times (x' \neq y') / 2 + (x'' = x') \times (y'' = y'))$$

$$= p^2 x' (x' \neq y') / 2 + p x (1-p) x (x' = \text{head}) \times (y' = \text{tail}) + (1-p) p x (x' = \text{tail}) \times (y' = \text{head}) + (1-p)^2 x' (x' \neq y') / 2$$

$$= (p^2 + 2 p x(1-p) + (1-p)^2) \times (x' \neq y') / 2$$

$$= R$$

For the timing, just put $t := t+1$ before the recursive call, and add timing to specification $R$:

$$R = (x' \neq y') \times (t' \geq t) \times (p^2 + (1-p)^2) t' - t \times p \times (1-p)$$

Here's the calculation.

$$\begin{cases} \text{if } p \text{ then } x := \text{head} \text{ else } x := \text{tail} \text{ fi} \\ \text{if } p \text{ then } y := \text{head} \text{ else } y := \text{tail} \text{ fi} \\ \text{if } x = y \text{ then } t := t+1. \ (x' \neq y') \times (t' \geq t) \times (p^2 + (1-p)^2) t' - t \times p \times (1-p) \text{ else } \text{ok fi} \end{cases}$$

$$= \Sigma x'', y'' \cdot t' \cdot (p x' = \text{head}) + (1-p) x' = \text{tail} \times (p y' = \text{head}) + (1-p) y' = \text{tail} \times ((x'' = y'') \times (t' \geq t+1) \times (x' \neq y') \times (p^2 + (1-p)^2) t' - t' - 1 \times p \times (1-p)$$

$$+ (x'' = x') \times (y'' = y') \times (t' = t)$$

$$= p^2 x' (t' \geq t+1) \times (p^2 + (1-p)^2) t' - t' \times p \times (1-p) + p x (1-p) x (x' = \text{head}) \times (y' = \text{tail}) \times (t' = t)$$
and the second equation is proved as follows:
\[
\begin{align*}
&+ (1-p) \times p \times (x' = \text{tail}) \times (y' = \text{head}) \times (t' = t) \\
&+ (1-p)^2 \times (x' = y') \times (t' \geq t+1) \times (p^2 + (1-p)^2) \times (t' = t) \\
&= (p^2 + (1-p)^2) \times (x' = y') \times (t' \geq t+1) \times (p^2 + (1-p)^2) \times (t' = t) \\
&+ p \times (1-p) \times (x' = y') \times (t' = t) \\
&= (x' = y') \times (t' \geq t+1) \times (p^2 + (1-p)^2) \times (t' = t) + p \times (1-p) \times (x' = y') \times (t' = t) \\
&= (x' = y') \times (t' \geq t) \times (p^2 + (1-p)^2) \times (t' = t) + p \times (1-p)
\end{align*}
\]
There was no need for an assumption that \( p \) differs from both 0 and 1 in either proof. But if \( p \) is either 0 or 1, the timing expression gives probability 0 to any finite value of \( t' \). And if \( p \) is either 0 or 1 we can easily prove \( t' = \infty \) (but we don't bother).

So the first program works. But the second program doesn't; it gives exactly the same result as a single flip of the coin. Here's the calculation. This time define
\[
R \iff \text{if } p \text{ then } x' = \text{head} \land y' = \text{tail} \text{ else } x' = \text{tail} \land y' = \text{head} \text{ fi}
\]
which is a single flip, and define
\[
S \iff x' = x+y'
\]
then the first equation is proved as follows:
\[
\begin{align*}
&\text{if } p \text{ then } x := \text{head} \text{ else } x := \text{tail} \text{ fi} \quad . \quad S \\
&= \text{if } p \text{ then } x := \text{head} \text{ else } x := \text{tail} \text{ fi} \quad . \quad x' = x+y' \\
&= \text{if } p \text{ then } x := \text{head} \quad . \quad x' = x+y' \text{ else } x := \text{tail} \quad . \quad x' = x+y' \text{ fi} \\
&= \text{if } p \text{ then } x' = \text{head} \text{ else } x' = \text{tail} + y' \text{ fi} \\
&= R
\end{align*}
\]
and the second equation is proved as follows:
\[
\begin{align*}
&\text{if } p \text{ then } y := \text{head} \text{ else } y := \text{tail} \text{ fi} \\
&\text{if } x = y \text{ then } x := y \text{ fi} \\
&\Sigma x', y' . \quad (p \times (x' = x) \times (y' = \text{head}) + (1-p) \times (x'' = x) \times (y'' = \text{tail})) \\
&\quad \times ((x' = y') \times (x' = x') \times (y' = y') + (x' = y'') \times (x' = x') \times (y' = y')) \\
&= p \times ((x = \text{head}) \times (x' = x) \times (y' = \text{head}) + (x + \text{head}) \times (x' = x) \times (y' = \text{head})) \\
&\quad + (1-p) \times ((x = \text{tail}) \times (x' = x) \times (y' = \text{tail}) + (x + \text{tail}) \times (x' = x) \times (y' = \text{tail})) \\
&= p \times ((x = \text{head}) \times (x' = x) \times (y' + x) + (x + \text{head}) \times (x' = x) \times (y' + x)) \\
&\quad + (1-p) \times ((x = \text{tail}) \times (x' = x) \times (y' + x) + (x + \text{tail}) \times (x' = x) \times (y' + x)) \\
&= (x' = x) \times (y' + x) \times (p \times (x = \text{head}) + (x + \text{head})) + (1-p) \times ((x = \text{tail}) + (x + \text{tail})) \\
&= (x' = x) \times (y' + x) \\
&= S
\end{align*}
\]