Let $n$ be natural and let $s$ be a natural variable. Using a for-loop, without using multiplication or exponentiation, write a program for $s' = n^2$.

After trying the question, scroll down to the solution.
I’ll use the invariant form of \texttt{for}-loop.

\[
\begin{align*}
& s’ = n^2 \iff s := 0. \quad A 0 \Rightarrow A’n \\
& A 0 \Rightarrow A’n \iff \texttt{for } k := 0..n \texttt{ do } k: 0..n \land A k \Rightarrow A'(k+1) \texttt{ od} \\
& k: 0..n \land A k \Rightarrow A'(k+1) \iff s := s + n
\end{align*}
\]

To complete the final refinement, we need to define $A k$. Here’s one way.

\[
\begin{align*}
& A k \quad s = k \times n \\
& \text{Now to change } k \times n \text{ into } (k+1) \times n \text{ we need to add} \\
& (k+1) \times n - k \times n = n \\
& \text{So we complete the solution:} \\
& k: 0..n \land A k \Rightarrow A'(k+1) \iff s := s + n
\end{align*}
\]

Here’s another way to define $A k$.

\[
\begin{align*}
& A k \quad s = k^2 \\
& \text{Now to change } k^2 \text{ into } (k+1)^2 \text{ we need to add} \\
& (k+1)^2 - k^2 = k+k+1 \\
& \text{So we complete the solution:} \\
& k: 0..n \land A k \Rightarrow A'(k+1) \iff s := s+k+k+1
\end{align*}
\]