327 (square) Let n be natural and let s be a natural variable. Using a **for**-loop, without using multiplication or exponentiation, write a program for $s'=n^2$.

After trying the question, scroll down to the solution.

§ I'll use the invariant form of **for**-loop.

$$s'=n^2 \quad \Leftarrow \quad s:=0.\ \ A\,0 \Rightarrow A'n$$

$$A\,0 \Rightarrow A'n \quad \Leftarrow \quad \textbf{for}\ k:=0;..n\ \textbf{do}\ k:\ 0,..n \wedge A\,k \Rightarrow A'(k+1)\ \textbf{od}$$

$$k:\ 0,..n \wedge A\,k \Rightarrow A'(k+1) \quad \Leftarrow \quad s:=\ s\ +$$

To complete the final refinement, we need to define $A\,k$. Here's one way.

$$A\,k \ = \ s = k\times n$$

Now to change $k\times n$ into $(k+1)\times n$ we need to add

$$(k+1)\times n - k\times n \ = \ n$$

So we complete the solution:

$$k:\ 0,..n \wedge A\,k \Rightarrow A'(k+1) \quad \Leftarrow \quad s:=\ s+n$$

Here's another way to define $A\,k$.

$$A\,k \ = \ s = k^2$$

Now to change k^2 into $(k+1)^2$ we need to add

$$(k+1)^2 - k^2 \ = \ k+k+1$$

So we complete the solution:

$$k:\ 0,..n \wedge A\,k \Rightarrow A'(k+1) \quad \Leftarrow \quad s:=\ s+k+k+1$$