319 Let p and n be natural variables. Prove

$$p'=2^{20} \Longleftarrow p:= 1.\ n:= 0.\ \textbf{while}\ n\ne 20\ \textbf{do}\ p:= p\times 2.\ n:= n+1\ \textbf{od}$$

After trying the question, scroll down to the solution.

§ I prove two refinements:

$p'=2^{20} \Leftarrow p:=1.\ n:=0.\ p' = p\times 2^{20-n}$

$p' = p\times 2^{20-n} \Leftarrow$ **if** $n\ne 20$ **then** $p:=p\times 2.\ n:=n+1.\ p' = p\times 2^{20-n}$ **else** ok **fi**

First refinement, right side:

	$p:=1.\ n:=0.\ p' = p\times 2^{20-n}$	substitution law twice
$=$	$p' = 1\times 2^{20-0}$	arithmetic
$=$	$p' = 2^{20}$	

Last refinement, right side:

	if $n\ne 20$ **then** $p:=p\times 2.\ n:=n+1.\ p' = p\times 2^{20-n}$ **else** ok **fi**	substitution law twice
$=$	**if** $n\ne 20$ **then** $p' = p\times 2\times 2^{20-(n+1)}$ **else** ok **fi**	arithmetic
$=$	**if** $n\ne 20$ **then** $p' = p\times 2^{20-n}$ **else** ok **fi**	expand ok
$=$	**if** $n\ne 20$ **then** $p' = p\times 2^{20-n}$ **else** $p'=p \land n'=n$ **fi**	arithmetic and context
$=$	**if** $n\ne 20$ **then** $p' = p\times 2^{20-n}$ **else** $p' = p\times 2^{20-n} \land n'=n$ **fi**	specialization, monotonicity
$\Longrightarrow$	**if** $n\ne 20$ **then** $p' = p\times 2^{20-n}$ **else** $p' = p\times 2^{20-n}$ **fi**	case idempotent
$=$	$p' = p\times 2^{20-n}$	