(nondeterministic assignment) Generalize the assignment notation $x := e$ to allow the expression $e$ to be a bunch, with the meaning that $x$ is assigned an arbitrary element of the bunch. For example, $x := \text{nat}$ assigns $x$ an arbitrary natural number. Show the standard binary notation for this form of assignment. Show what happens to the Substitution Law.

After trying the question, scroll down to the solution.
\[ x := e \quad \Rightarrow \quad x' : e \land y' = y \land \ldots \]

\[ x := e. \quad P \]
\[ \equiv \quad \exists x', y', \ldots (x': e \land y' = y \land \ldots) \land \text{(substitute } x', y', \ldots \text{ for } x, y, \ldots \text{ in } P) \]
\[ \equiv \quad \exists x': x': e \land \text{(substitute } x' \text{ for } x \text{ in } P) \]

but the one-point law does not allow us to get rid of \( \exists x' \). For example, in one variable,

\[ x := 0, 1. \quad x' = x + x \]
\[ \equiv \quad \exists x''. \quad x'' : 0, 1 \land \text{(substitute } x'' \text{ for } x \text{ in } x' = x + x) \]
\[ \equiv \quad \exists x''. \quad x'' : 0, 1 \land x' = x'' + x'' \]
\[ \equiv \quad x' = 0 + 0 \lor x' = 1 + 1 \]
\[ \equiv \quad x' : 0, 2 \]

but the Substitution Law would give

\[ \equiv \quad x' = (0, 1) + (0, 1) \]
\[ \equiv \quad x' = 0, 1, 2 \]