303 Let x be an integer state variable, and there are no other state variables.

(a) For what exact precondition does $x:= x^2$ make x be even?

(b) What does it mean to say that your answer to part (a) is the exact precondition for $x:= x^2$ to make x be even?

(c) For what exact postcondition does $x:= x^2$ make x be even?

(d) What does it mean to say that your answer to part (c) is the exact postcondition for $x:= x^2$ to make x be even?

After trying the question, scroll down to the solution.

(a) For what exact precondition does $x := x^2$ make x be even?

§ (the exact precondition for $even\ x'$ to be refined by $x := x^2$)

$= \quad \forall x' \cdot even\ x' \Leftarrow (x := x^2)$ assignment

$= \quad \forall x' \cdot even\ x' \Leftarrow x' = x^2$ one-point

$= \quad even\ x^2$ arithmetic

$= \quad even\ x$

(b) What does it mean to say that your answer to part (a) is the exact precondition for $x := x^2$ to make x be even?

§ It means that squaring x makes x be even if and only if x was even already.

(c) For what exact postcondition does $x := x^2$ make x be even?

§ (the exact postcondition for $even\ x'$ to be refined by $x := x^2$)

$= \quad \forall x \cdot even\ x' \Leftarrow (x := x^2)$ assignment

$= \quad \forall x \cdot even\ x' \Leftarrow x' = x^2$ antidistributive law

$= \quad even\ x' \Leftarrow \exists x \cdot x' = x^2$ bunch-element conversion

$= \quad even\ x' \Leftarrow x' : int^2$

$= \quad even\ x' \lor \neg x' : int^2$

(d) What does it mean to say that your answer to part (c) is the exact postcondition for $x := x^2$ to make x be even?

§ It means that squaring x makes x be even if and only the final value of x is even or the final value if x is not a square. Well, the final value of x will be a square, so I guess that just means squaring x makes x even if and only if the final value of x is even, which is not very helpful.