For what exact precondition and postcondition does the following assignment move integer variable $x$ farther from zero?

(a) $x := x + 1$

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(the exact precondition for $\text{abs } x' > \text{abs } x$ to be refined by $x := x + 1$)

\[ \forall x': \text{abs } x' > \text{abs } x \iff (x := x + 1) \]

\[ \forall x': \text{abs } x' > \text{abs } x \iff x' = x + 1 \]

\[ \text{abs } (x + 1) > \text{abs } x \]

\[ x \geq 0 \]

(the exact postcondition for $\text{abs } x' > \text{abs } x$ to be refined by $x := x + 1$)

\[ \forall x: \text{abs } x' > \text{abs } x \iff (x := x + 1) \]

\[ \forall x: \text{abs } x' > \text{abs } x \iff x' = x + 1 \]

\[ \text{abs } (\text{abs } (x + 1)) > \text{abs } x \]

\[ x' \geq 1 \]

(b) $x := \text{abs } (x + 1)$

§

(the exact precondition for $\text{abs } x' > \text{abs } x$ to be refined by $x := \text{abs } (x + 1)$)

\[ \forall x': \text{abs } x' > \text{abs } x \iff (x := \text{abs } (x + 1)) \]

\[ \forall x': \text{abs } x' > \text{abs } x \iff x' = \text{abs } (x + 1) \]

\[ \text{abs } (\text{abs } (x + 1)) > \text{abs } x \]

\[ x \geq 0 \]

(the exact postcondition for $\text{abs } x' > \text{abs } x$ to be refined by $x := \text{abs } (x + 1)$)

\[ \forall x: \text{abs } x' > \text{abs } x \iff (x := \text{abs } (x + 1)) \]

\[ \forall x: \text{abs } x' > \text{abs } x \iff x' = \text{abs } (x + 1) \]

\[ \text{abs } (\text{abs } (x + 1)) > \text{abs } x \]

\[ x' \geq 0 \]

(c) $x := x^2$

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(the exact precondition for $\text{abs } x' > \text{abs } x$ to be refined by $x := x^2$)

\[ \forall x': \text{abs } x' > \text{abs } x \iff x' = x^2 \]

\[ \text{abs } (x^2) > \text{abs } x \]

by the arithmetic properties of $\text{abs } x$ and $x^2$

\[ x \neq -1 \land x \neq 0 \land x + 1 \]

(One-Point Law)

\[ \text{arithmetic: } x^2 = (\text{abs } x)^2 \]

\[ (\text{the exact postcondition for } \text{abs } x' > \text{abs } x \text{ to be refined by } x := x^2) \]

\[ \forall x: \text{int } \text{abs } x' > \text{abs } x \iff x' = x^2 \]

\[ \forall x: \text{int } \text{abs } x' > \text{abs } x \iff x' = (\text{abs } x)^2 \]

change variable
\[ \forall y: \text{abs} \rightarrow \text{abs} \quad x^\prime > y \iff x^\prime = y^2 \]

\[ \forall y: \text{nat} \quad \text{abs} (y^2) > y \iff x^\prime = y^2 \]

\[ \forall y: \text{nat} \quad y^2 > y \iff x^\prime = y^2 \]

\[ (\forall y: 0 \quad y^2 > y \iff x^\prime = y^2) \land (\forall y: 1 \quad y^2 > y \iff x^\prime = y^2) \land (\forall y: \text{nat} + 2 \quad y^2 > y \iff x^\prime = y^2) \]

\[ (\bot \iff x^\prime = 0) \land (\bot \iff x^\prime = 1) \land (\forall y: \text{nat} + 2 \quad \top \iff x^\prime = y^2) \]

\[ x^\prime \neq 0 \land x^\prime \neq 1 \]